SWITCHED PIECEWISE UNIFORM VECTOR QUANTIZATION OF THE MEMORYLESS TWO-DIMENSIONAL LAPLACIAN SOURCE IN A WIDE DYNAMIC RANGE OF POWER

Zoran H. PERIC, Vladimir DESPOTOVIC
Department of Telecommunications, Faculty of Electronic Engineering,
University of Nis, Aleksandra Medvedeva 14, 18000 Nis, tel. (+381)18/529225, E-mail: peric@elfak.ni.ac.yu

SUMMARY

In this paper a simple and complete asymptotical analysis is given for a switched piecewise uniform quantization of two-dimensional memoryless Laplacian source with the respect to distortion (D) i.e. the mean-square error (MSE). Piecewise uniform quantization consists of L different uniform vector quantizers. Uniform quantizer optimality conditions and all main equations for optimal number of levels and constant (nonoptimal) number of output points for each partition are presented (using rectangular cells). Switched quantization is used in order to give higher quality by increasing signal-to-quantization noise ratio (SNRQ) in a wide range of signal volumes (variances) or to decrease necessary sample rate. These systems, although not optimal, may have asymptotic performance arbitrarily close to the optimum. Further more, their anaysis and implementation can be simpler than those of optimal systems.

Keywords: *Piecewise Uniform Quantization, Switched Quantization, Granular Distortion, Overload Distortion*

1. INTRODUCTION

The use of digital representation for audio, speech, images and video is rapidly growing with the extended use of computers and multimedia computer applications. To provide a more efficient representation of data, many compression algorithms have been developed, and in the basis of all these algorithms is quantization. The concept of quantization is a mapping of a large set of amplitudes of infinite precision to a smaller finite set of values.

The quantizers play an important role in the theory and practice of modern signal processing. The asymptotic optimal quantization problem, even for the simplest case — uniform scalar quantization, is actual nowadays [1, 2]. They do consider the problem of finding the optimal maximum amplitude, so-called, support region for scalar quantizers by minimization of the total distortion D, which is a combination of granular (D_g) and overload (D_o) distortion, $D = D_g + D_o$.

The analysis of vector quantizer for arbitrary distribution of the source signal is given in paper [3]. The authors derived the expression for the optimum granular distortion and optimum number of output points. However, they did not prove the optimality of the proposed solutions. Also, they did not define the partition of the multidimensional space into subregions. In paper [4], the expressions for the optimum number of output points are derived, however the proposed partitioning of the multidimensional space for memoryless Laplacian source does not consider the geometry of the multidimensional source. In paper [5], vector quantizers of Laplacian and Gaussian sources are analyzed. The proposed solution for the quantization of memoryless Laplacian source, unlike in [5], takes into consideration the geometry of the source, however, the proposed vector quantizer design procedure is too complicated and unpractical.

In this paper we will find signal to quantization noise ratio of suggested switched piecewise uniform vector quantizer (PUQ) for Laplacian memoryless source, for a case of constant number of output points on each domain. Indeed, we want to suggest the quantizer that, compared to the one with optimal number of output points, may have asymptotic performance arbitrarily close to the optimum. We will give a general and simple way to design a switched piecewise uniform vector quantizer. We will derive the optimal number of levels for each partition and the optimality of the proposed solutions is proved. The goal of this paper is to solve a quantization problem in a case of PUQ and to find corresponding support region. It is done by analytical optimization of the granular distortion and numerical optimization of the total distortion. If the distortion is measured by a squared error, D becomes the mean squared error (MSE). The distortion mean-squared error (MSE i.e. quantization noise) is used as a criterion for optimization.

The MSE of a two-dimensional vector source $x = (x_1, x_2)$, where x_i are zero-mean statistically independent Laplacian random variables of variance σ^2, is commonly used for the transform coefficients of speech or imagery. The first approximation to the long-time-averaged probability density function (pdf) of amplitudes is provided by Laplacian model [6, p. 32]. The waveforms are sometimes represented in terms of adjacent-sample differences. The pdf of the difference signal for an image waveform follows the Laplacian function [6, p.33]. The Laplace source is a model for speech [7, p.384]. Consider two independent identically distributed Laplace random variables (x_1, x_2) with the zero mean. To simplify the vector quantizer, the Helmert transformation is applied on the source vector giving contours with constant probability densities. The transformation is defined as:

$$r = \frac{1}{\sqrt{2}}\left(\left|x_1\right| + \left|x_2\right|\right), \quad u = \frac{1}{\sqrt{2}}\left(\left|x_1\right| - \left|x_2\right|\right) \tag{1}$$

In this paper, quantizers are designed and analysed under additional constraint — each scalar quantizer is a uniform one.

PUQ consists of L optimal uniform vector quantizers. More precisely, our quantizer divides the input plane into L partitions and every partition is further subdivided into L_i ($1 \le i \le L$) subpartitions. Every concentric subpartition can be subdivided in four equivalent regions, i.e. the j-th subpartition in signal plane is allowed to have p_{ij} ($1 \le i \le L, 1 \le j \le L_i$) cells. We perform distortion optimization (D_i) in every partition under the constraint:

$$4\sum_{j=1}^{L_i} p_{ij} = N_i \tag{2}$$

In this work we design a piecewise uniform vector quantizer for optimal compression function. We perform analytical optimisation of the granular distortion and numerical optimization of the total distortion using rectangular cells.

2. SWITCHED TWO-DIMENSIONAL VECTOR QUANTIZATION MODEL

During the two dimensional vector quantization, vector obtained by sampling of input signal in two adjacent points is replaced with vector from allowed set of vectors in such way that the quantization error is the smallest. Successful vector quantization depends on an appropriate choice of allowed vector set (codebook). The nearest neighbor quantizer completely searches codebook. If the codebook is size of N ($N=2^{16}$ for R=8 bit/sample, $N=2^{15}$ for R=7.5 bit/sample), then N distortion estimation would be needed. In the common communication systems for the typical large vector rates requires enormous number of arithmetic operations in second, i.e. processors faster than 10^{13} operations in second. That is the reason for the serious study of more effective algorithms that give code vector of nearest neighbor without the complete search of codebook. Some solutions are based on vector quantizers which have codebook of special structure so the encoding is faster. On the other hand, the price is the suboptimal codebook and insignificantly weaken performances.

The switching quantization aims are to improve the quality of the signal-to-noise ratio in the wide range of the signal average power (i.e. variance) or to decrease the sample rate. The switching quantization is adaptive quantization for memoryless sources and it is aplicable only if adaptation is performed on the basis of the signal average power, what was done in this paper. As an input source, we will consider memoryless Laplacian source.

Let us asume that input signal has constant power (σ^2 = *const.*). In practice the cases with constant power are not very often. For example, when we are taking into consideration the speech, known fact is that the average speech power varies from man to man. So we will classify middle loud, loud and quiet orators group. In this case we need infinity number of quantizers designed for all values σ from the considered range (adaptive quantization).

The presented problem of average power random nature we will solve with switched quantization application. The basic scheme of switched codebook adaptation is shown in Figure 1. One simple technique is switched codebook adaptive vector quantization. This technique uses a classifier that looks at the contents of the input frame buffer and decides that the next block of vectors belongs to a particular statistical class of vectors from a finite set of K possible classes. Namely, the index specifying the class is used to select a particular codebook from a predesigned set of K codebooks. This index is also transmitted as side information to the receiver. Then each vector in the block is encoded by the vector quantizer which performs a search through the selected codebook.

One block is made of M vectors. The index to identify the class is sent on the end of block, while the index to identify the codebook is sent with each vector. If each of the K codebooks has N code vectors, then the bit rate per sample is:

$$R_0 = \frac{1}{n}\log_2 N + \frac{1}{n}\frac{\log_2 K}{M} = R + \Delta R \tag{3}$$

where n is quantizer dimension. The second term in equation (3) is due the side information [7].

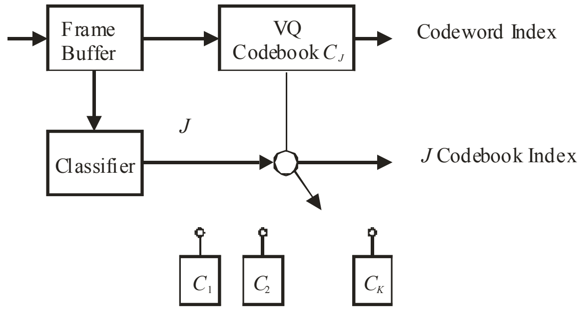

Fig. 1 Switched codebook adaptive vector quantization

We will use this technique for our problem solving. In our case the quantizer dimension is $n = 2$. We have K codebooks, i. e. K piecewise uniform quantizers designed for particular value σ_{0j} and for the cover of particular input power range $\sigma^2 \in [\sigma_{1j}^2, \sigma_{2j}^2)$, where $\sigma_{1j} < \sigma_{0j} < \sigma_{2j}$ and :

$$\bigcup_{j=1}^{K} \left[\sigma_{1j}^2[dBm], \sigma_{2j}^2[dBm]\right) = [-47, -3] \tag{4}$$

Also all codebooks are of the same size N. For elected value σ_{0j}, $j = 1, 2, ..., K$ we will design piecewise uniform vector quantizers. We should determine distortion for input Laplacian source which power is $\sigma^2 \in [\sigma_{1j}^2, \sigma_{2j}^2)$, while the quantizer is designed in such way that it has the smallest distortion for the power σ_{0j}^2.

3. OPTIMALITY CONDITIONS FOR UNIFORM TWO-DIMENSIONAL QUANTIZATION AND QUANTIZER DESIGN

Joint pdf function of two independent, identically distributed Laplace random variables (x_1 , x_2) with zero mean is given with the following expression

$$f_{1,2}(x_1, x_2) = \frac{1}{2\sigma^2} e^{-\frac{\sqrt{2}(|x_1|+|x_2|)}{\sigma}} \tag{5}$$

To simplify the vector quantizer, the Helmert transformation is applied on the source vector, giving contours with constant probability densities. This orthogonal transformation is defined as [8,9]:

$$r = \frac{1}{\sqrt{2}}(|x_1| + |x_2|), \quad u = \frac{1}{\sqrt{2}}(|x_1| - |x_2|) \tag{6}$$

The obtained probability density function is:

$$f(r,u) = \frac{1}{2\sigma^2} e^{-\frac{2r}{\sigma}} \tag{7}$$

In the two-dimensional ru system the pdf function given by equation (7) represents a square line. The square surface (0, $r_{\max}$) representing dynamic range of a two-dimensional quantizer, can be partitioned into L concentric domains as shown in Fig. 2. The number of output points in each domain is denoted by N_i, where $N = \sum_{i=1}^{L} N_i$ represents the total number of output points. Every concentric domain can be further partitioned into L_i concentric subdomains of equal width. Every subdomain is divided into four regions each containing $p_{i,j}$ rectangular cells. An output point is placed in the centre of each cell. Coordinates of the k-th output point in j-th subregion of the i-th region in the ru coordinate system are $\left(m_{i,j}, \hat{u}_{i,j,k}\right)$.

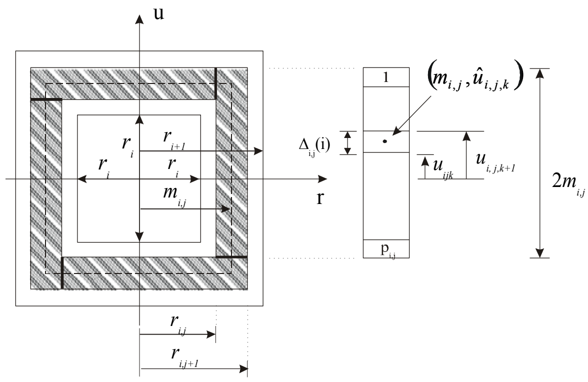

Fig. 2 Two-dimensional space partitioning

The quantising cells are rectangular and the representation vector is $c_{ijk} = \left(m_{i,j}, \hat{u}_{i,j,k}\right)$. The quality of a quantizer can be measured by the goodness of the resulting reproduction in comparation to the original. One way to accomplish this is to define a distortion measure d that quantifies cost or distortion, and to consider average distortion as a measure of the quality of a system, with smaller average distortion meaning higher quality. The most common and widely used measure of distortion between an input vector r and quantised vector c_{ijk} is the squared error [7, p. 325] Euclidean distance between two vectors defined as:

$$d = (r, c_{ijk}) = \left|r - c_{ijk}\right|^2 = \left(r - m_{ij}\right)^2 + \left(u - \hat{u}_{ijk}\right)^2 \tag{8}$$

The initial expression for granular distortion is

$$D_g = 4\sum_{i=1}^{L}\sum_{j=1}^{L_i}\sum_{k=1}^{p_{i,j}} \int_{r_{i,j}}^{r_{i,j+1}} \int_{u_{i,j,k}}^{u_{i,j,k+1}} \left[(r - m_{i,j})^2 + (u - \hat{u}_{i,j,k})^2\right] \cdot \frac{1}{2\sigma^2} e^{-\frac{2r}{\sigma}} dr du \tag{9}$$

The output point coordinates are given by the equations

$$m_{i,j} = \frac{r_{i,j+1} + r_{i,j}}{2}, \quad \hat{u}_{i,j,k} = \frac{u_{i,j,k} + u_{i,j,k+1}}{2} \tag{10}$$

Rectangular cell dimensions are:

$$\Delta = r_{i+1} - r_i = r_{\max} / L, \quad \Delta_i = \frac{\Delta}{L_i}$$

$$\Delta_{ij} = \frac{r_{i,j} + r_{i,j+1}}{p_{i,j}} = \frac{2m_{i,j}}{p_{i,j}} \tag{11}$$

$$r_i = i\Delta;\;\; r_{i,j} = r_i + j \cdot \Delta_i$$
$$i = 0,\dots,L\,,\;\; j = 0,\dots,L_i \tag{12}$$

The range of the quantizer is $r_{\max}$. The total number of output points is

$$N = \sum_{i=1}^{L} N_i \tag{13}$$

where N_i is the number of output points in the i-th domain. We can also write:

$$\sum_{j=1}^{L_i} p_{i,j} = \frac{N_i}{4} \tag{14}$$

Equation (9) can be written as:

$$D_g = \sum_{i=1}^{L} D_g(i) \tag{15}$$

where $D_g(i)$ is:

$$D_g(i) = 4\sum_{j=1}^{L_i}\sum_{k=1}^{p_{i,j}} \int_{r_{i,j}}^{r_{i,j+1}} \int_{u_{i,j,k}}^{u_{i,j,k+1}} \left[(r - m_{i,j})^2 + (u - \hat{u}_{i,j,k})^2\right] \cdot \frac{1}{2\sigma^2} e^{-\frac{2r}{\sigma}} dr du \tag{16}$$

After integration over u and reordering equation (16) becomes

$$D_g(i) = \sum_{j=1}^{L_i} [\int_{r_{i,j}}^{r_{i,j+1}} (r - m_{i,j})^2 (r_{i,j+1} + r_{i,j}) \frac{2}{\sigma^2} e^{-\frac{2r}{\sigma}} dr + \int_{r_{i,j}}^{r_{i,j+1}} \frac{(r_{i,j+1} + r_{i,j})^3}{12 p_{i,j}^{\;2}} \frac{2}{\sigma^2} e^{-\frac{2r}{\sigma}} dr] \tag{17}$$

From equation (10) it follows that $r_{i,j+1} + r_{i,j} = 2m_{i,j}$. When we substitute this in equation (17) we get

$$D_g(i) = \sum_{j=1}^{L_i} [\int_{r_{i,j}}^{r_{i,j+1}} (r - m_{i,j})^2 4m_{i,j} \frac{1}{\sigma^2} e^{-\frac{2r}{\sigma}} dr + \int_{r_{i,j}}^{r_{i,j+1}} \frac{m_{i,j}^{\;3}}{3p_{i,j}^{\;2}} 4m_{i,j} \frac{1}{\sigma^2} e^{-\frac{2r}{\sigma}} dr] \tag{18}$$

We will now assume that $\exp(-2r/\sigma)$ is constant over Δ_i. In that case we can substitute $\exp(-2r/\sigma)$ with $\exp(-2m_{i,j}/\sigma)$. Equation (18) can be now written as:

$$D_g(i) = \sum_{j=1}^{L_i} 4m_{i,j} \frac{1}{\sigma^2} e^{-\frac{2m_{i,j}}{\sigma}} \left[\int_{r_{i,j}}^{r_{i,j+1}} (r - m_{i,j})^2 dr + \int_{r_{i,j}}^{r_{i,j+1}} \frac{m_{i,j}^2}{3p_{i,j}^2} dr\right]$$

$$= \sum_{j=1}^{L_i} 4m_{i,j} \frac{1}{\sigma^2} e^{-\frac{2m_{i,j}}{\sigma}} \left[\frac{\Delta_i^3}{12} + \frac{m_{i,j}^2}{3p_{i,j}^2}\Delta_i\right] =$$

$$= \sum_{j=1}^{L_i} \left[2m_{i,j}\frac{\Delta_i^2}{12} + \frac{2m_{i,j}^3}{3p_{i,j}^2}\right] \cdot P(m_{i,j}) \tag{19}$$

where $P(m_{i,j})$ denotes the probability

$$P(m_{i,j}) = \Delta_i f(m_{i,j}) = \Delta_i \frac{2}{\sigma^2} e^{-\frac{2m_{i,j}}{\sigma}} \tag{20}$$

Function $f(m_{i,j})$ is defined as

$$f(m_{i,j}) = \frac{2}{\sigma^2} e^{-2\frac{m_{i,j}}{\sigma}} \tag{21}$$

By using the Langrangian multipliers we can obtain the optimum number of cells in one region $p_{i,j}$, which yields the minimum granular distortion defined by the equation (19). Because we are designing an optimal quantizer for one value of variance σ_0, in calculating $p_{i,j}$ we will use σ_0 instead of σ. We will start from the following equation:

$$J = D_g(i) + \lambda\sum_{j=1}^{L_i} p_{i,j} \tag{22}$$

If we substitute $D_g(i)$ from equation (19) in equation (22) we obtain:

$$J = \sum_{i=1}^{L_i} \left[2m_{i,j}\frac{\Delta_i^2}{12} + \frac{2m_{i,j}^3}{3p_{i,j}^{\;2}}\right] \cdot P_0(m_{i,j}) + \lambda\sum_{j=1}^{L_i} p_{i,j} \tag{23}$$

After differentiating J with respect to $p_{i,j}$, and equalizing the derivate with zero we get

$$p_{i,j} = \frac{N_i}{4} \frac{m_{i,j}\sqrt[3]{g_0(m_{i,j})}}{\sum_{k=1}^{L_i} m_{i,k}\sqrt[3]{g_0(m_{i,k})}} \tag{24}$$

where $g_0\left(m_{i,j}\right)$ denotes the function

$$g_0\left(m_{i,j}\right)=\frac{1}{\sigma_0^2}e^{-\frac{2m_{i,j}}{\sigma_0}} \tag{25}$$

If we multiply numerator and denominator with Δ_i, we can approximate the sum by the integral

$$p_{i,j}=\frac{N_i}{4}\frac{m_{i,j}\sqrt[3]{g_0\left(m_{i,j}\right)}\cdot\Delta_i}{\int_{r_i}^{r_{i+1}} r\cdot\sqrt[3]{g_0(r)}\,dr} \tag{26}$$

By substituting $p_{i,j}$ from equation (26) in equation (19) we get

$$\begin{aligned}D_g(i)=&\frac{\Delta_i^2}{3}\sum_{j=1}^{L_i} m_{i,j}g_0(m_{i,j})\Delta_i+\\&+\frac{64}{3N_i^2\Delta_i^2}I_0'(i)^2\sum_{j=1}^{L_i} m_{i,j}\frac{g(m_{i,j})}{g_0(m_{i,j})^{2/3}}\Delta_i\end{aligned} \tag{27}$$

After approximating the sum by the integral, we can rewrite (27) as

$$D_g(i)=\frac{\Delta^2}{3L_i^2}I(i)+\frac{64\,L_i^2}{3N_i^2\Delta^2}I_0'(i)^2\,I'(i) \tag{28}$$

The functions $I_0'(i)$, $I'(i)$ and $I(i)$ are defined as:

$$\begin{aligned}I_0'(i)&=\int_{r_i}^{r_{i+1}} r\cdot\sqrt[3]{g_0(r)}\,dr;\\I'(i)&=\int_{r_i}^{r_{i+1}} r\cdot\frac{g(r)}{g_0(r)^{2/3}}\,dr;\\I(i)&=\int_{r_i}^{r_{i+1}} r\cdot g(r)\,dr.\end{aligned} \tag{29}$$

After integration over r and we obtain:

$$I_0'(i)=-\frac{9\sigma_0^{4/3}}{4}-e^{-\frac{2i\Delta}{3\sigma_0}}\left[\left(1-e^{\frac{2\Delta}{3\sigma_0}}\right)\left(1+\frac{2i\Delta}{3\sigma_0}\right)+\frac{2\Delta}{3\sigma_0}e^{\frac{2\Delta}{3\sigma_0}}\right]$$

$$\begin{aligned}I(i)&=-\frac{1}{4}e^{-2i\Delta/\sigma}\left[\left(1-e^{2\Delta/\sigma}\right)\left(1+2i\Delta/\sigma\right)+2\Delta/\sigma e^{2\Delta/\sigma}\right]\\I'(i)&=-\frac{9}{4}\left(\frac{\sigma_0^{5/3}}{3\sigma_0-2\sigma}\right)^2-e^{-\frac{2i\Delta}{3\sigma'}}\cdot\\&\cdot\left[\left(1-e^{\frac{2\Delta}{3\sigma'}}\right)\left(1+\frac{2i\Delta}{3\sigma'}\right)+\frac{2\Delta}{3\sigma'}e^{\frac{2\Delta}{3\sigma'}}\right]\end{aligned} \tag{30}$$

where $\sigma'=\dfrac{\sigma_0\sigma}{3\sigma_0-2\sigma}$.

After differentiating D_g from equation (28) with respect to L_i, and for σ_0 and equalizing the derivate with zero $\frac{\partial D_g(i)}{\partial L_i}=0$, we obtain the optimum number subdomains in i-th domain

$$L_{iopt}=\Delta\sqrt[4]{\frac{I_0(i)N_i^2}{64I_0'(i)^3}} \tag{31}$$

where $I_0(i)$ is defined as

$$\begin{aligned}I_0(i)&=\int_{r_i}^{r_{i+1}} r\cdot g_0(r)\,dr=\\&-\frac{1}{4}e^{-2i\Delta/\sigma_0}\left[\left(1-e^{2\Delta/\sigma_0}\right)\left(1+2i\Delta/\sigma_0\right)+2\Delta/\sigma_0 e^{2\Delta/\sigma_0}\right]\end{aligned}$$

Substituting the expression for L_i from equation (31) in equation (28), $D_g(i)$ becomes

$$D_g(i)=\frac{8}{3N_i}I_0'(i)\left(\sqrt{\frac{I_0'(i)}{I_0(i)}}\cdot I(i)+\sqrt{\frac{I_0(i)}{I_0'(i)}}\cdot I'(i)\right) \tag{32}$$

We will now assume that the number of output points in the ith subdomain N_i is constant under the constraint $N=\sum_{i=1}^{L}N_i=2^{2R}$, where R is bit rate (number of bits per sample). If we assume that bit rate is $R=R_1+R_2$, where $2R_1$ bits are used for representing the number of levels L, we can find R_1 from $L=2^{2R_1}$. The number of output points in the ith subdomain N_i is:

$$N_i=2^{2R_2}=2^{2(R-R_1)}=2^{2R-\frac{\log L}{\log 2}}=const. \tag{33}$$

By substituting N_i from preceding equation in equation (32) we get granular distortion. Now, we can calculate the total granular distortion of uniform piecewise vector quantizer as

$$D_g=\sum_{i=1}^{L_i}D_g(i)$$

We can calculate the overload distortion as

$$D_0=4\sum_{j=1}^{p_{L,L_L}}\int_{r_{\max}}^{\infty}\int_{u_{L,j}}^{u_{L,j}}\left[\left(r-m_{L,L_L}\right)^2+\left(u-\hat{u}_{L,j}\right)^2\right]\frac{1}{2\sigma^2}e^{-\frac{2r}{\sigma}}\,dr\,du$$

After some calculation, we get:

$$D_o = \frac{m_{L,L_L}}{\sigma^2} e^{-\frac{2r_{\max}}{\sigma}} \left[2\sigma r_{\max}^2 + r_{\max}\left(2\sigma^2 - 4\sigma m_{L,L_L}\right) + \right. \tag{34}$$

$$\left. + \sigma^3 - 2m_{L,L_L}\sigma^2 + 2m_{L,L_L}^2\sigma + \sigma\frac{2m_{L,L_L}^2}{3p_{L,L_L}^2}\right].$$

The total distortion for one dimension is:

$$D = \frac{1}{2}(D_g + D_o)$$

Since we now know how to calculate distortion for piecewise uniform vector quantizer (PUQ) for Laplacian memoryless source, we can find signal power to total distortion ratio in dB. SNRQ is a signal-to-quantization noise ratio, given with:

$$SNRQ[dB] = 10\log\frac{\sigma^2}{Duk} \tag{35}$$

4. NUMERICAL RESULTS

In this section we will show the results obtained for signal to quantization noise ratio of the switched piecewise uniform vector quantization for Laplacian memoryless source. The results shown in Fig. 3 present signal to quantization noise ratio as a function of input power for special case of switched quantizer (K = 8, for eight values of σ_0 i.e. eight different quantizers), the bit rate of R = 7,5 bit per sample, for L=8 concentric domains and for constant number of output points in the ith subdomain N_i :

$$\sigma_0 \text{ [dBm]} = -40+j5, \text{ where } j = 0,\ldots,7$$

In formula above, σ_0 represents variance on the basis of which the quantizer is designed. Ni can be calculated from equation (33). Constant number of output points in the ith subdomain N_i = 4096.

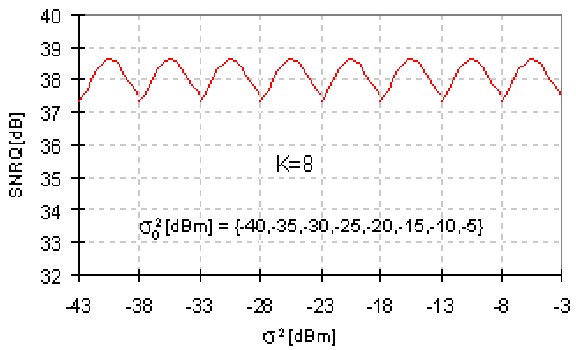

Fig. 3 SNRQ for bit rate R = 7,5 , N_i= 4096, R_0 = 7.525, M=60 and for eight quantizers designed for different values of σ_0

We can see that SNRQ will never fall under 37 dB. In the case of using a switched quantizer with more than eight quantizers, performance could be even better.

In Fig. 4 we can see signal-to-quantization noise ratio for optimum number of output points and for eight quantizers designed for different values of σ_0. Optimum number of output points N_i is calculated in [9]

$$N_i = N\frac{\left[I_0'(i)^3 I_0(i)\right]^{1/4}}{\sum_{k=1}^{L}\left[I_0'(k)^3 I_0(k)\right]^{1/4}} \tag{36}$$

where $N = \sum_{i=1}^{L} N_i = 2^{2R} = 32768.$

Maximum value of SNRQ is about 2dB better than in a case we have constant number of output points N_i , but the curve decreased slower, so the difference between minimum values in this two cases is greater then 2.5dB. Nevertheless, we can conclude that these systems, although not optimal, may have asymptotic performance arbitrarily close to the optimum. Further more, their anaysis and implementation can be much simpler than those of optimal systems.

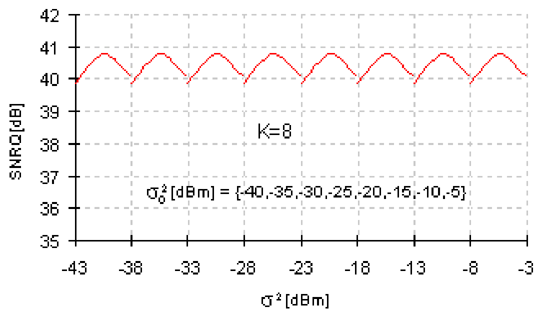

Fig. 4 SNRQ for bit rate R = 7,5 , R_0 = 7.525, M=60 optimum number of output points N_i and for eight quantizers designed for different values of σ_0

Figure 5 shows SNRQ for bit rate R_0 = 7.025 bps and for constant number of output points in the ith subdomain. It is obvious that switched quantizers that encode sample with 7.025 bits have about 3 dB less SNRQ than corresponding quantizers that encode sample with 7.525 bits. Hence, in cases when the larger SNRQ is important, we will choose quantizer with larger number of bits per sample.

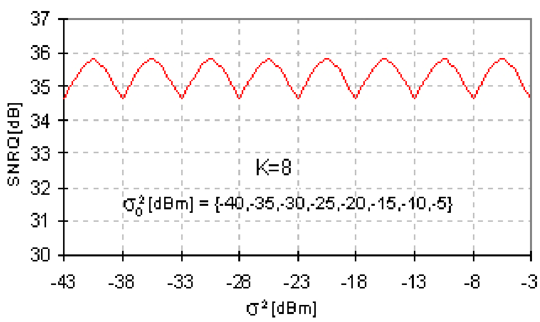

Fig. 5 SNRQ for bit rate R = 7, R_0 = 7.025, M=60N_i=const and for eight quantizers designed for different values of σ_0

In systems with switched quantizers, each vector belongs to particular class of K possible classes and the information about that must be transmitted. For K classes representation, $\log_2 K$ bits (in our case 3 bits) are necessary, because in such way each code combination uniformly signs one class, i. e. one vector quantizer and its codebook. During the switched quantizer design the particular memory is needed. In cases when the memory resources are limited, it is possible to decrease the vector classes number K, but SNRQ will have larger variation due to input power changes.

5. CONCLUSION

We suggested a model of switched piecewise vector quantizer which solves problem of variable input power in a wide range. We have showen that this switched quantizer can be applied for speech signals that have not only the random nature of instantaneous signal values, but also the random nature of the average power. A simple expression for granular distortion, a number of subdomains and a number of output points in closed form is obtained. Memoryless Laplacian source is used, considering the possible application of this quantizer. The results are obtained by using switched quantization with eight vector quantizers optimized for eight different values of σ (variance), in order to get a better compression quality with higher SNRQ.

REFERENCES

[1] Hui D., Neuhoff D.L: Asymptotic Analysis of Optimal Fixed-Rate Uniform Scalar Quantization., IEEE Trans. 2001, IT-47(3), pp. 957–977.

[2] Na S., Neuhoff D.L.: On the Support of MSE-Optimal, Fixed-Rate Scalar Quantizers, IEEE Trans. Inform. Theory 2001, IT-47(6), pp. 2972–2982.

[3] Kuhlmann F., Bucklew J.A..: Piecewise Uniform Vector Quantizers, IEEE Trans. 1988., IT-34(5), pp. 1259–1263.

[4] Swaszek P.F.: Unrestricted Multistage Vector Quantizers, IEEE Trans. 1992., IT-38(3), pp. 1169–1174.

[5] Jeong D.G., Gibson J.: Uniform and Piecewise Uniform Lattice Vector Quantization for Memoryless Gaussian and Laplacian Sources, IEEE Trans. 1993, T-39(3), pp. 786–804.

[6] Jayant N.S.and Noll P.: Digital coding of waveforms - Principles and Applications to Speech and Video, New Jersey: Prentice-Hall, 1984.

[7] Gersho A. and Gray R.M.: Vector Quantization and Signal Compression, Kluwer: Academ.Pub, 1992.

[8] Zoran H. Peric, Jelena D. Jovkovic, Zoran J. Nikolic.: Two-Dimensional Laplas Source Quantization, IEEE Conf. TELSIKS 2001. Nis (Serbia), pp. 33–37.

[9] Z.H.Peric, A.Jovanovic, S.Bogosavljevic: Optimal Piecewise Uniform Vector Quantization of the Memoryless Two Dimensional Laplacian source, Third IEEE Workshop on "Inteligent Data Acquisition and Advanced Computing Systems": Technology and Applications Sofia, Bulgaria 5-7 Septembar, (2005) pp. 540-546.

BIOGRAPHIES

Zoran H. Perić was born in Nis, Serbia, in 1964. He received the B. Sc. degree in electronics and telecommunications from the Faculty of Electronic science, Nis, Serbia, Yugoslavia, in 1989, and M. Sc. degree in telecommunication from the University of Nis, in 1994. He received the Ph. D. degree from the University of Nis, also, in 1999. He is currently Professor at the Department of Telecommunications, University of Nis, Serbia. His current research interests include the information theory, source and channel coding and signal processing. He is particularly working on scalar and vector quantization techniques in compression of images. He has authored and coauthored over 90 scientific papers.

Vladimir Despotović was born in Zenica, Bosnia and Herzegovina, in 1978. He received the B.Sc. degree in telecommunications from the Faculty of Electronic science, Nis, Serbia in 2003. He is currently student of the postgraduate studies at the Department of Telecommunications, University of Nis, Serbia. He is working as a system engineer in Global Scada Services, Belgrade, Serbia. His current research interests include the speech signal processing and process control. He is particularly working on scalar and vector quantization techniques in compression of speech signal. He has authored and coauthored 10 scientific papers.