USING HYBRID SYSTEMS THEORY TO SIMULATE THE BEHAVIOUR OF STEP-DOWN DC-DC CONVERTERS

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ABSTRACT

This paper presents a simulation technique for step-down DC-DC converters based on the hybrid systems theory. The proposed technique is able to simulate the behaviour of the converter in both transient and steady state regimes. To show the applicability of the proposed technique a buck converter is considered. Both output voltage and inductor current are acquired with a digital oscilloscope and are compared with the ones obtained with the proposed technique during steady state regime to show its applicability. Besides, both transient and steady responses of the converter obtained from the proposed simulation technique are compared with ones computed from MATLAB/SIMULINK environment.

Keywords: *Simulation, modelling, hybrid models and switch mode power converters.*

NOMENCLATURE

V_{in} : input voltage;
R_L : inductor resistance;
L : inductor inductance;
R_S : drain-source resistance;
V_d : diode forward voltage drop;
R_d : diode internal resistance;
C : capacitor capacitance;
ESR : capacitor internal resistance;
R : load resistance;
i_L : inductor current;
i_C : capacitor current;
i_O : load current;
v_O : output voltage.

1. INTRODUCTION

There are mainly three kinds of electronic power conversion devices, and they can be classified according to input and output voltage as: *DC/DC* converters, *AC/DC* converters and *DC/AC* converters (or inverters). This paper deals only with the *DC/DC* power converters.

The *DC-DC* power converters can be classified in linear or switch mode types. The first ones convert the input voltage into a lower output voltage by dissipating power through an active component (usually a bipolar transistor operating in linear mode); while the second ones use a power transistor (e.g. Mosfets) operating in both cut-off and saturation region, which makes them much more efficient than the first ones.

Over time the demand of power supplies physically small, more efficient, with stability over a wide range of inputs, with several outputs and lighter, turn the switch mode power supplies, *SMPS*, the obvious choice for manufacturers [1]. *SMPS* can be classified in isolated or non-isolated types. The first ones use a high frequency transformer that isolates electrically the load from the main source, while the second ones are not electrically isolated. This paper will focus on the second ones, to be more exact on the *buck* converter.

The development of user friendly simulation techniques, which can be implemented easily in an open source platform for numerical computation (e.g. *scilab*) is quite important for design purposes. These simulation programs can avoid wasting of time and money. For instance, new design ideas can be tested and improved using the simulation software without burning electronic equipment or wasting time constructing new prototypes. Besides, they are particularly vital whenever the required measurement equipment is unavailable (e.g. high-voltage floating probe).

2. BUCK CONVERTER

The *buck* converter is composed by two semiconductors (a diode and a transistor) and two reactive elements: an inductor and a capacitor, as illustrated in Fig. 1. It can operate in two conduction modes: continuous conduction mode (*CCM*) and discontinuous conduction mode (*DCM*). The second one will rise if during steady state regime, in a switching period, the inductor current reaches zero.

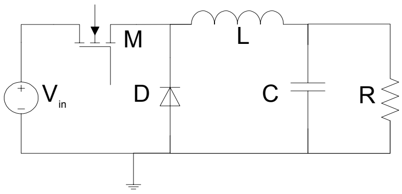

Fig. 1 *Buck* converter schematics

During the conduction stage, the transistor, *M*, turns on and the diode, *D*, is reverse biased, so the current will pass through the inductor, *L*, which stores energy in the form of a magnetic field, and simultaneously the capacitor, *C*, is charged, Fig. 2a.

During the non-conduction stage the transistor is turn off which forces the current to free wheel around the path consisting of *L*, *C* and *D*, Fig. 2b.

During the transistor turn off, if the inductor current reaches zero a new stage will rise: the discontinuous stage. In this stage the capacitor will supply power to the load, Fig. 2c.

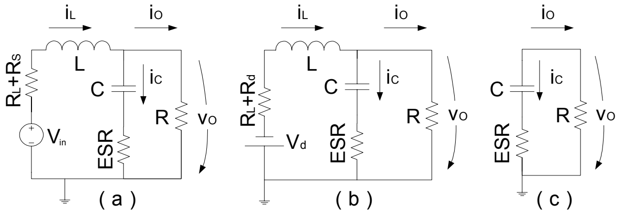

Fig. 2 Equivalent circuit of *buck* converter: (a) conduction stage; (b) non-conduction stage; and (c) discontinuous stage.

The state space representation is a mathematical model of a physical system. The inputs, outputs and state variables are related by first-order differential equations. So the variables are expressed as vectors and the differential and algebraic equations are written in a matrix form, which provides a compact way to model and analyze the system:

$$\dot{x} = A\,x + b\,u \tag{1}$$

where,

x : vector of the state variables;
u : vector of independent sources (e.g. input voltage);
A, B : system matrices.

Using Fig. 2 and seeing that each state is a linear and time invariant system it is possible to represent each state in the form of equation (1). So, for the conduction stage:

$$\begin{bmatrix} \dfrac{di_L}{dt} \\ \dfrac{dv_O}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} i_L \\ v_O \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ \dfrac{ESR \times R}{L \times (R + ESR)} \end{bmatrix} V_{in} \tag{2}$$

$$a_{11} = -\frac{R_S + R_L}{L}; a_{12} = -\frac{1}{L};$$

$$a_{21} = \frac{R}{C \times (R + ESR)} - \frac{ESR \times R \times (R_L + R_S)}{L \times (R + ESR)}$$

$$a_{22} = \frac{-1}{C \times (R + ESR)} - \frac{ESR \times R}{L \times (R + ESR)}$$

For the non-conduction stage:

$$\begin{bmatrix} \dfrac{di_L}{dt} \\ \dfrac{dv_O}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} i_L \\ v_O \end{bmatrix} + \begin{bmatrix} -\dfrac{1}{L} \\ -\dfrac{ESR \times R}{L \times (R + ESR)} \end{bmatrix} V_d \tag{3}$$

$$a_{11} = -\frac{R_d + R_L}{L}; a_{12} = -\frac{1}{L};$$

$$a_{21} = \frac{R}{C \times (R + ESR)} - \frac{ESR \times R \times (R_L + R_d)}{L \times (R + ESR)}$$

$$a_{22} = \frac{-1}{C \times (R + ESR)} - \frac{ESR \times R}{L \times (R + ESR)}$$

For the discontinuous stage:

$$\begin{bmatrix} \dfrac{di_L}{dt} \\ \dfrac{dv_O}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{-1}{C \times (R + ESR)} \end{bmatrix} \begin{bmatrix} i_L \\ v_O \end{bmatrix} \tag{4}$$

Despite each state represent a linear system the converter itself is a non-linear system. Therefore, using a method such as the space state averaging technique (*SSA*) the discontinuity associated with the transitions between states can be smoothed. For that, a linearization process around the stable operating point can lead to a set of continuous linear equations, with which it is possible to obtain the transient response and frequency response of the converter [2].

The proposed simulation technique allows the determination of both transient and steady response of the converter.

3. POWER CONVERTER HYBRID MODEL

A *buck* converter is a hybrid dynamic system. It can be described as set of discrete states with associated continuous dynamics.

Let $X \subseteq R^n$ be a continuous state space and $Q = \{q_1, q_2, \ldots, q_N\}$ be a finite set of discrete states, the continuous dynamics can be modeled by differential equations [3]:

$$\dot{x} = f_q\big(x(t)\big) \tag{5}$$

where,

$x \in X$: vector of the state variables;
$q \in Q$: the on/off configuration of the all switches.

If $s \in R^m$ and represents the on/off sate of the m switches in the circuit the hybrid model can be described as [3]:

$$\dot{x} = \hat{A}\,x(t) + \hat{B}\,s(t) + C + \sum_{i=1}^{m} s_i(t)\,\hat{H}_i\,x(t) \tag{6}$$

where, $\hat{A} : R^{nxn}, \hat{B} : R^{nxm}, C : R^n, \hat{H}_i : R^{nxn}$, $i = 1,2, \ldots, m$ and $s_i(t)$ represents the component switch vector s. The value of s_i is 0 or 1.

The hybrid model of the *buck* converter can represented as:

$$\begin{bmatrix} \dfrac{di_L}{dt} \\ \dfrac{dv_O}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} i_L \\ v_O \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} i_L \\ v_O \end{bmatrix} (s_1 + s_2) + \tag{7}$$

$$\begin{bmatrix} c_{11} & 0 \\ c_{21} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_O \end{bmatrix} s_1 + \begin{bmatrix} d_{11} & 0 \\ d_{21} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_O \end{bmatrix} s_2 + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} s_1 + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} s_2$$

$$a_{22} = \frac{-1}{C \times (ESR + R)}; b_{11} = \frac{-R_L}{L}; b_{12} = \frac{-1}{L};$$

$$b_{21} = \frac{R}{C \times (R + ESR)} - \frac{R \times ESR \times R_L}{L \times (R + ESR)}; b_{22} = \frac{-ESR \times R}{L \times (R + ESR)};$$

$$c_{11} = \frac{-R_S}{L}; c_{12} = \frac{-ESR \times R \times R_S}{L \times (R + ESR)}; d_{11} = \frac{-R_d}{L};$$

$$d_{12} = \frac{-ESR \times R \times R_d}{L \times \left(R + ESR\right)};\ e_1 = \frac{V_{in}}{L};\ e_2 = \frac{ESR \times R \times V_{in}}{L \times \left(R + ESR\right)};$$

$$f_1 = -\frac{V_d}{L};\ f_2 = -\frac{ESR \times R \times V_d}{L \times \left(R + ESR\right)}$$

where s_1 is a vector that represents the conduction stage, so it is one during the conduction stage and zero during the non-conduction and discontinuous stages. The vector s_2 represents the non-conduction stage.

By, representing (7) in its discrete form, it is possible to develop a simulation program for the *buck* converter. Following, a small algorithm is presented:

```
$i = 1;$
while $i < NTP$
if $\left(i_L(i) > 0 \text{ and } s_1(i) > 0\right)$
$\rightarrow i_L(i+1) = i_L(i) + (T\,b_{11}) \times i_L(i) \times \left(s_1(i) + s_2(i)\right) +$
$(T\,b_{12}) \times v_O(i) \times \left(s_1(i) + s_2(i)\right) + (T\,c_{11}) \times i_L(i) \times s_1(i) +$
$(T\,d_{11}) \times i_L(i) \times s_2(i) + (T\,e_1) \times s_1(i) + (T\,f_1) \times s_2(i);$
$\rightarrow v_O(i+1) = v_O(i) + (T\,a_{22}) \times v_O(i) +$
$(T\,b_{21}) \times i_L(i) \times \left(s_1(i) + s_2(i)\right) + (T\,b_{22}) \times v_O(i) \times \left(s_1(i) + s_2(i)\right) +$
$(T\,c_{21}) \times i_L(i) \times s_1(i) + (T\,d_{21}) \times i_L(i) \times s_2(i) +$
$(T\,e_2) \times s_1(i) + (T\,f_2) \times s_2(i);$
else
$\rightarrow i_L(i+1) = i_L(i);$
$\rightarrow v_O(i+1) = v_O(i) + (T\,a_{22}) \times v_O(i);$
end
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where *NTP* represents the total number of iterations and is defined by the user.

4. SIMULATED RESULTS

In order to judge the applicability of the proposed technique some simulated results were computed, and are compared with the simulation technique proposed in [4] and the simulated results obtained from *MATLAB/SIMULINK* environment.

Table 1 shows the characteristics of the prototypes used:

Table 1 Prototypes characteristics.

Converter	C_1	C_2	C_3
V_{in}	12 V		
L	22 μH		
R_L	0.14 Ω		
R_S	0.07 Ω		
R_d	0.01 Ω		
V_d	1.1 V		
D	0.3	0.5	0.7
T	50 μs		
C	1000 μF		
ESR	69 mΩ		
R	1 Ω		

Figs. 3, 4 and 5 show both waveforms of inductor current and output voltage, during transient and steady regime for converters C_1, C_2 and C_3, respectively.

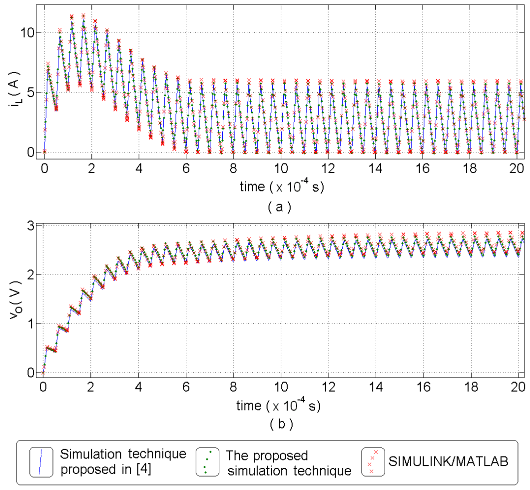

Fig. 3 Converter C_1 waveforms obtained from simulation techniques: (a) inductor current and (b) output voltage.

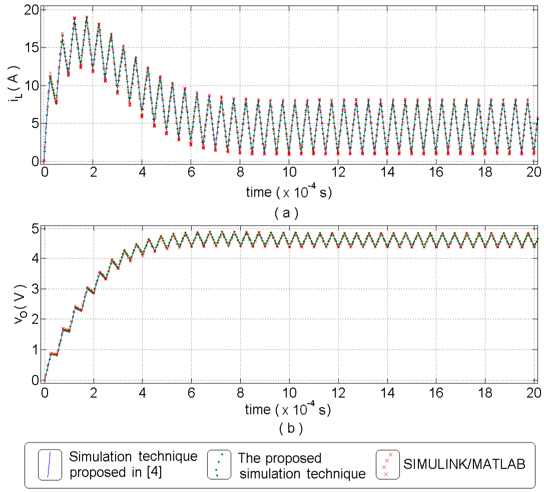

Fig. 4 Converter C_2 waveforms obtained from simulation techniques: (a) inductor current and (b) output voltage.

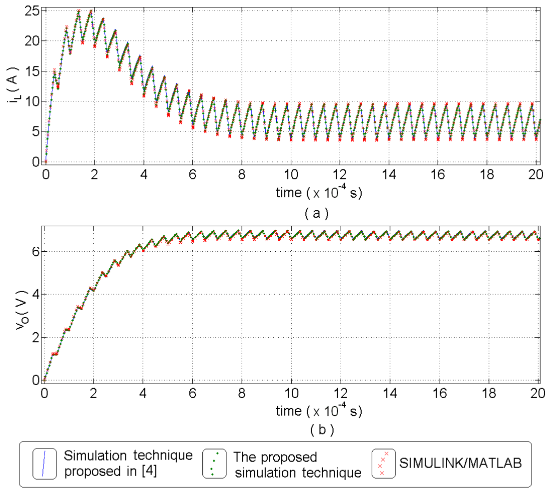

Fig. 5 Converter C_3 waveforms obtained from simulation techniques: (a) inductor current and (b) output voltage.

The simulation waveforms obtained from the different simulation techniques are almost undistinguishable. Thus, it is possible to conclude that the new proposed simulation technique can be used with much profit, as the technique proposed in [4], or using *MATLAB/SIMULINK*.

5. EXPERIMENTAL RESULTS

In this section the steady state waveforms of inductor current and output voltage ripple obtained with the proposed technique are compared with the experimental ones. For that, the prototypes characterized in Table I were designed. The experimental data was acquired through a digital oscilloscope *Tektronix TDS 1012*.

The diode, Mosfet and inductor equivalent circuits were obtained from the manufacturers specifications. However, for the capacitor equivalent circuit it was necessary to use the off-line technique proposed in [5], since its *ESR* changes considerable with both temperature and frequency.

Figs. 6-8 show a comparison between simulated and experimental results for converters C_1, C_2 and C_3, respectively.

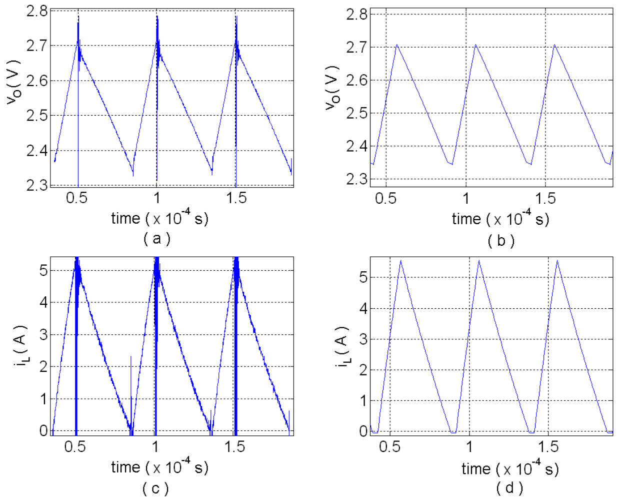

Fig. 6 Experimental (a,c) and simulated (b,d) results: (c,d) inductor current and (a,b) output voltage ripple, of C_1.

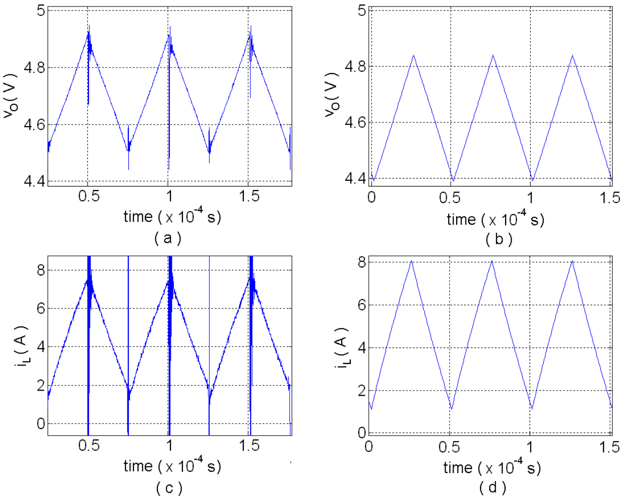

Fig. 7 Experimental (a,c) and simulated (b,d) results: (c,d) inductor current and (a,b) output voltage ripple, of C_2.

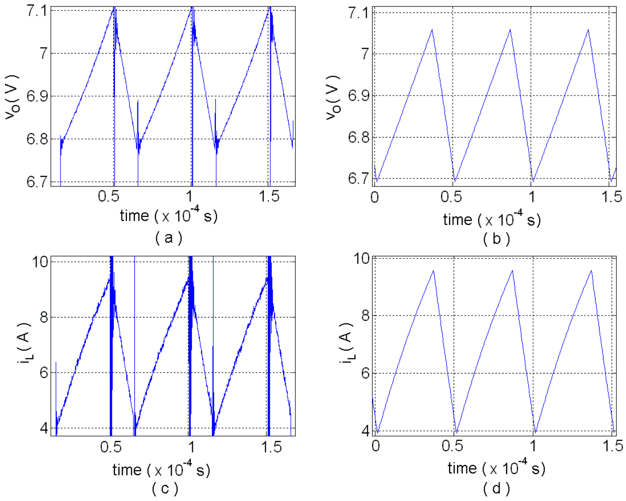

Fig. 8 Experimental (a,c) and simulated (b,d) results: (c,d) inductor current and (a,b) output voltage ripple, of C_3.

From Figs. 6-8 it is possible to conclude that both experimental and simulated results are very close, so the proposed technique can be used with much profit.

6. CONCLUSIONS

In this paper a simulation technique based on hybrid systems theory was presented and applied to a step-down *DC-DC* converter. This paper showed that the proposed technique is able to simulate the behaviour of the converter in both transient and steady state regime. The main advantage of the proposed technique is its simplicity and the possibility of being implemented in an open source platform for numerical computation, avoiding the use of heavy and very expensive simulation tools.

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BIOGRAPHIES

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