MODELISATION AND OPTIMIZATION OF SQUIRREL-CAGE USING ORTHOGONAL DESIGNS

*Naimi MOKHTARI, *Abdelkader ZEBLAH, **Abdelkader LOUSDAD, **Yamani MASSIM
*Department of Electrical Engineering, University of Sidi Bel Abbes Algeria
**Department of Mechanical Engineering, University of Sidi Bel Abbes Algeria

SUMMARY

In terms of general approach the problem of optimal design and construction is considered. In the present paper an optimization method is elaborated and takes on the process in establishing mathematical models for conducting real procedure that allows an optimal design and construction of asynchronous electrical motor of 13kW. The proposed method is a multi-objective optimization approach that determines the most influencing parameters and there interaction effects on both the economical function and the desired performances using design of experiment. The Strategy adopted avoids cumbersome computations used in traditional optimization techniques.

Keywords: *optimization, DOE, orthogonal design, strategy, real process, decision variable, output function*

1. INTRODUCTION

In the literature concerned with the problems of optimal construction of induction motors many mathematical optimization models have been used in order to determine the most influencing parameters on the objective function and on the constraint functions with their critical values. The optimal construction is formulated as a non linear problem in [3],[4] and [10]. The authors use the sequential unconstrained minimization techniques (SUMT) to obtain the minimum by transforming the constrained problem in a form such that the numerical solutions are obtained by solving a series of unconstrained problems using penalty methods. The use of these methods for obtaining the minimum cost has disadvantages as mentioned in [1],[8] due to the fact that the convergence is excessively difficult and it does frequently converges toward a sub optimal point. On the other hand in [2],[8] and [9] the authors consider the optimal construction as a non linear programming and use constrained minimization technique (CMT). The strategy adopted for obtaining models by these techniques is based on the study of the variation effects of one parameter at a time on the economical function and on the constrained functions. The repercussions of this approach is that it hides the interaction effects which might be very influencing and does not allow the quantification of factors effects. From these works we can retain that the major factors maintained to establish the models are: the diameter D_a, the thickness of the air gap E, the induction in the air gap B, the stator and rotor current densities respectivelly J_s and J_r. The objective function and the constrained functions are: the cost of active materials, the overload capacity $C_{\max}/C_d$, the ratio of the starting and nominal currents I_d / I_n, the temperature T, the power factor $\mathrm{Cos}(\varphi)$, the efficiency η the ratio of the starting and nominal torques C_d / C_n. In the present paper the computational procedure for caged motors is elaborated such that the normalizing constraints and those imposed by the specifications are strictly respected and that the simultaneous variations of the factors can be made with a planed and previously defined manner according to the desired design. This approach allows from on one hand, to reduce considerably the number of experiments and the allocated computational time and, on the other hand, allows to quantify the factors effects and the interactions on the objective function and on the constrained functions. It also permits to increase the number of factors and to elaborate simple regression models based on design of experiment (DOE).

2. REAL PROCESS AND MODEL STRATEGY RELATIONSHIP

In many real phenomena with complex interactions we are enable to get sufficient information on the evolution or on the state of the system through deterministic theoretical models with complex computations. We rely on simulating the laws of probability taken for the behavior of the components of the phenomenon under certain hypothesis and numerically compute a certain number of states. If we consider a phenomenon characterized by a scalar it can be submitted to the action of many scalar variables we say that intervenes many variables or alternatives called decision variables associated to the product construction. The value of each variable represents the associated level of activity. The problem that arises is how to know if the assumed action of variables is effective or not and if so what will be the relationship between the phenomenon and the considered variables which will require an experimentation. That is a series of experiments

This work was supported in part by the laboratory IRECOM and Department of Electrical Engineering University of Sidi Bel Abbes Algeria. (mokhtari@mail.univ-sba.dz)

during which different values are given to the variables under consideration in order to know the assumed influences of these variables on the scalar characterizing the phenomenon, this serie is finite and discrete. As described through this aspect the problem can be schematized by the following system fig. 1:

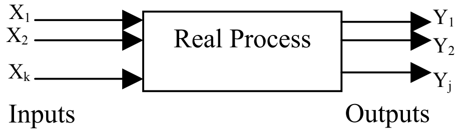

Fig. 1 Black box

The strategy adopted to obtain a feasible model is summarized in the following five steps:

1. To take into consideration all the phenomenon;
2. To know how to isolate, to measure and to be able to characterize them by a scalar;
3. To collect the numerical data according to the prescribed design;
4. To elaborate a theory capable of processing numerical results (strategy of the model);
5. Confronting the elaborated theory to the facts and deduct the consequences (analysis). This step is the aim of the test theory or generally mathematical statistical which covers all the aspects undertaken by statistical decision taken from numbers.

The flow-chart below fig. 2 shows the five main processes used to determine the relationship between the real process and the model. The processes III, IV, V are formalized and processes I and II are not.

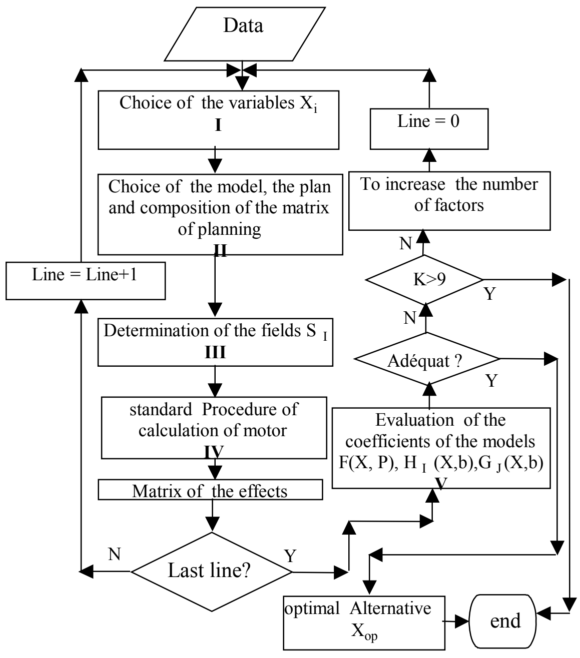

Fig. 2 Flow-chart

The objective function is approximated by $F(X,b)$, the equality constraints by $H_i\,(X,b)$ and the inequality constraints by $G_j\,(X,b)$.

3. CHOICE OF THE DECISION VARIABLES

The process initially consists in the counting of the variables X_i capable of influencing the system and secondly in ranking the variables according to their degree of influence, in order to be able, if it is the case with a higher number of variables, to eliminate those with a lesser influence. The variables with a lower influence are maintained constant during the experimental tests. The quantitative variables X_i chosen to describe the system evolution are considered as first order decision variables and are the only ones that appear in the analytical expressions of the system model. To reach this objective we tried to analyze the answers to the following questions:

1. What are the active materials?
2. What are the quantities that characterize the use of these materials?
3. What are the relationships between these characteristics and the geometrical quantities of the motor?
4. How they influence the cost function and the output function (constraint functions), when we vary one variable at a time and when we simultaneously vary all the variables X_i ?

Besides the original aspect of this article on the study of the effects of the simultaneous and planned variation of all the variables on the output functions, the experience held in the manufacturing domain of electrical machineries widely answers our questions. The high number of variables leads us to study the problem of the design of electrical motors not as a model of deterministic knowledge but as a strategy model of the system " Decision Making Problem". Based on these considerations we have maintained and classified the variables X_i as follows: External diameter D_a, the linear density A, the statoric current densities J_s and rotoric J_r and the thickness of air-gap E. The imposed constraints in the appraisal and the performances interested in are the efficiency η and the power factor Cos(φ). The ratio of the torques $C_d\,/\,C_n$ and the overload capacity $C_{\max}\,/\,C_n$ (K_{max}). The ratio of the starting current with respect to the nominal current $I_d\,/\,I_n$ and the temperature T (in continuous nominal regime) S1. To justify this choice we have studied the influence of these variables on the economical function and the influence on the performance of the motor. The graphical representation gives a good illustration of this dependency.

4. INFLUENCE OF THE INPUT VARIABLES ON THE OUTPUT FUNCTIONS

Inspiring from the invariance theory of forms by variation of size (scale theory), we have ponderated the output and input quantities in such a manner that the values are of the same order for the sake to be able to represent all the characteristics in the same graph and to limit the space for investigation in searching the optimal combination. Weighting is made according to relation (9). The table I gives an indication on the accepted scales. (52D.A equal one US Dollar).

	Cost DA	η	Cos(φ)	K_{max}	T (C°)
min	2200.00	0.885	0.830	2.055	47.00
max	2630.00	0.905	0.890	3.055	57.00
X_{iB}	2415.00	0.895	0.860	2.555	52.00
ΔX_i	215.00	0.010	0.030	0.500	5.00

Table I Symbols and levels of output parameters, coded values

4.1. Economical function (cost of active materials)

We note that when increasing separately the factors A, J_s, J_r reduces the cost in opposition to the increase of D_a and E which leads to an increase fig. 3 and fig. 4.

We do not observe the same effect when varying simultaneously these factors. This analysis leads us to conclude that the effect of D_a and E taken together is less important than the effect induced by A, J_s, J_r taken together.

4.2. Efficiency

The factors D_a, A, J_s, J_r have a negative influence on the efficiency when they are taken separately. The effect of the air-gap is not significant. The conjugate effect of the simultaneous variation of all the variables leads to an increase of the efficiency in the interval [-1.5, 0.5] which is opposite to the effect when taking the factors separately. The maximum is reached at $X = 0.5$

4.3. Power factor

The power factor increases when J_s and J_r increase and decreases with E . We observe that it reaches a maximum when A and D_a increase then decreases afterwards.

The same effect is observed on the graph in fig. 5. This means that the power factor is more sensitive to the variation of A and D_a then to that of J_s, J_r and E taken together.

4.4. Ratio of the starting torque to the nominal torque and Overload capacity

C_d/C_n increases with J_r and slightly increases with J_s , an increase of the factors D_a and A leads to an increase of this ratio. The noticeable effect is due to the variation of the air-gap in fact, the ratio highly decreases and goes through a minimum corresponding to $X = 0.25$ then increases, but the sensitivity to the increase of E is not important then the decrease of the ratio. The overload capacity increases with J_r and E, and decreases with D_a, A and J_s. The effect of each one is significant.

4.5. Temperature

The effect of these factors on the temperature is opposite to their effects on the efficiency which is reasonable.

The Temperature is more sensitive to the effect of the current density J_s than the interaction effect of A and J_s .

4.6. Effect of the simultaneous variations of X_i on Y_j

The shapes of the curves obtained show the importance of the interactions effects on the output functions. The effects of the factors shown in the figures 3,4,5,6 and 7 as with a less influence on certain output functions are induced with errors due to the maintaining of constant parameters. According to the objective aimed, we arrive by this representation to canalize and reduce the space of investigation for the search of optimum option. When making an increasing variation of all the variables simultaneously we observe that:

1. The shape of the curve of the overload capacity can be assimilated to a constant affined line with a lower shape (polynomial form of lower order less than one).
2. The curve of the temperature has the tendancy of a convex parabolic form of higher order greater than one.
3. The forms of the curves representing the cost, the efficiency and the power factor have the tendancy of concave parabolic forms of order greater than one (polynomial form of higher order greater than one).

From these observations we can deduct the adequate type of the approximation model.

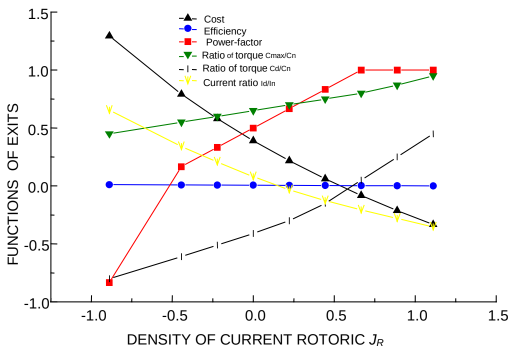

Fig. 3 Influence of the rotorique current density on the output parameters

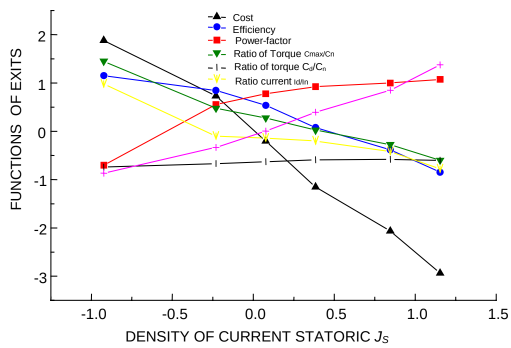

Fig. 4 Influence of the statorique current density on the output parameters

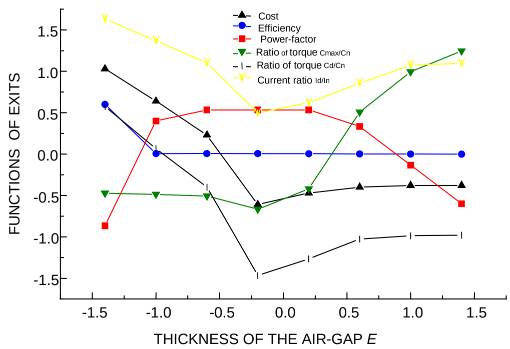

Fig. 5 Influence of the thickness of the air-gap on the output parameters

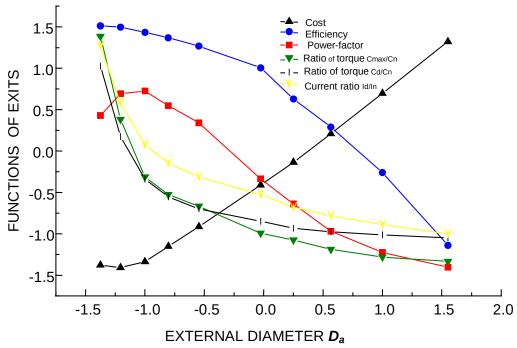

Fig. 6 Influence of the outside diameter on the output parameters

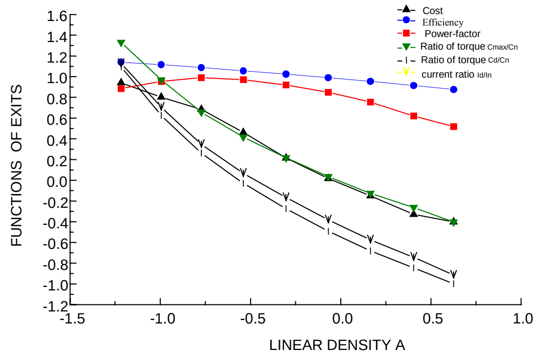

Fig. 7 Influence of the linear density on the output parameters

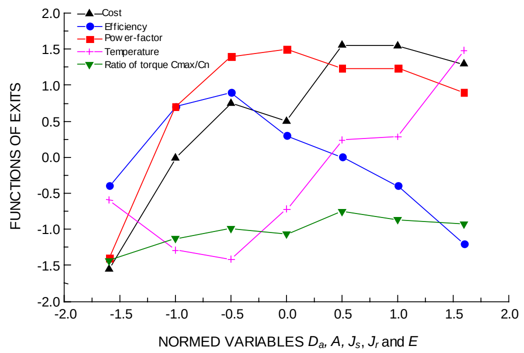

Fig. 8 Influence of simultaneous variation of all the variables on the output parameters

5. CHOICE OF THE DESIGN AND MODEL

5.1. The Model

The effectiveness and the accuracy of any model depend on the appropriate data on the subject and the guess of the investigator. Our objective is to choose the more precise model that transmits, in quantity and quality, information of major importance that allow to minimize the cost of active materials and improve motor efficiency. From this point of view we avoid to substitute the interactions between factors by new factors and more precisely the interaction between the linear density and the statoric current AJ_s that indicate the losses. We need to establish the linking equations between the output characteristics and the factors influencing each one of theses. The equations are assimilated to links between the output signals of the system represented in fig. 1 and the input variables $\vec{X} = (X_1, X_2, ..., X_K)$ which are accessible to permanent observation, are under the form

$$Y = \nu + \varepsilon = f(\vec{X}) + \varepsilon \qquad (1)$$

Where ε is an unobservable random signal under normal conditions and whose statistical characteristics satisfy the following conditions:

– The process is statistically independent of $\vec{X}$

- It is centered
- Has a limited dispersion
- It is ergodic with respect to its mean value $f(\vec{X})$ and is an unknown operator which determines the dependency of Y and $\vec{X}$. In the case of regular object $v=f(\vec{X})$ depends on the time t_0 if it is a component of the vector $\vec{X}$

The development of the linking equations between the output parameters Y_i and the input parameters $X_1, X_2, \ldots, X_k$ are seeked under the form of polynomial of a Taylor series segment the operator $f(\vec{X})$ can be approximated by.

$$F(X,b)=\sum_{|\beta|\leq n} b_\beta X[1]^{\beta^i}\, X[2]^{\beta^i}\, X[k]^{\beta^i} \tag{2}$$

$|\beta|$ Length of the multi index

β_i Exponents of order i (β_i= 0, 1, 2, ..., n)

$\beta = (\beta_1, \beta_2, \beta_3, \ldots, \beta_k)$

$|\beta| = \beta_1 + \beta_2 + \beta_3 + \ldots + \beta_k$ et $|\beta| \leq n$

Relation (2) is the Taylor development of order N of $F(X,b)$

For the problems developed in this paper, we have used models of order less or equal to two (02) in this case the regression equation has the following form:

$$\tilde{Y}=\tilde{b}_0+\sum_{i=1}^{k}\tilde{b}_i X_i+\sum_{\substack{i,j\\ j<i}}^{k}\tilde{b}_{ij} X_i X_j+\sum_{i=1}^{k}\tilde{b}_{ii} X_i^2 \tag{3}$$

Where $\tilde{Y}$: is the effective value of the output random parameter

X_i, X_j: Corresponding input variables

$\tilde{b}_0, \tilde{b}_i, \tilde{b}_{ij}, \tilde{b}_{ii}$: effective values of the coefficients of the equation.

The repercussions of the effects of the non controllable input variables (disturbances) of the statistical properties of the object on one hand and those of the correlative links between controllable and non controllable factors, with the errors of the input and output data on the other hand, make that the link determined by (3) is not strictly functional due to the real dependency between the input and output parameters. This relation is statically correlative it is established between the output mean values of the object Y and the current values of the inputs and is put under the form:

$$\overline{Y}=f(\vec{X})=b_0+\sum_{i=1}^{k} b_i X_i+\sum_{\substack{i,j\\ j<i}}^{k} b_{ij} X_i X_j+\sum_{i=1}^{k} b_{ii} X_i^2 \tag{4}$$

where: b_0, b_i, b_{ij}, b_{ii} are estimators of the coefficients of the regression equation. The coefficients of (4) can be determined by the solutions of the system of linear equations. This can be done easily under matrix form, these equation (4) can be written as:

$$\boldsymbol{Y}=f^T(\boldsymbol{X}).\boldsymbol{B} \tag{5}$$

$$f^T(\boldsymbol{X})=\left|f_0(\boldsymbol{X}), f_1(\boldsymbol{X}), \ldots; f_k(\boldsymbol{X})\right|=\left|X_{0u}, X_{1u}, \ldots, X_{ku}\right| \tag{6}$$

Where $f^T(\boldsymbol{X})$ is the transpose matrix of the elements $f_\gamma(\boldsymbol{X})$ $(\gamma=\overline{0,k})$ and $\boldsymbol{B}$ is the matrix of estimators of coefficients. By introducing fictitious variable $X_0=1$, and by putting $X_{k+1}=X_1^2$; $X_{k+2}=X_2^2$; $X_{2k}=X_k^2$; $X_{2k+1}=X_1X_2$; $X_{2k+2}=X_1X_3$; …; $X_{2k+C_k^l}=X_1X_2\ldots X_k$

These equation (4) can be reduced to the following linear homogeneous equation.

$$\boldsymbol{X}.\boldsymbol{B}=\boldsymbol{Y} \tag{7}$$

Where $\boldsymbol{X}$ is a rectangular matrix of the observed values; $\boldsymbol{Y}$ is a column matrix of the output variable. C_k^l Is the number of all the possible combinations of k elements to l $(l=\overline{1,k})$.

The solution of equation (7) is :

$$B=(X^TX)^{-1}X^TY \tag{8}$$

$$\left[\boldsymbol{X}^T\boldsymbol{X}\sigma^2(Y)\right]^{-1}=\begin{vmatrix}\sigma^2(b_0) & \operatorname{cov}(b_0b_1) & \cdots & \operatorname{cov}(b_0b_k)\\ \operatorname{cov}(b_1b_0) & \sigma^2(b_0) & \cdots & \operatorname{cov}(b_1b_k)\\ \vdots & \vdots & \vdots & \vdots\\ \operatorname{cov}(b_kb_0) & \operatorname{cov}(b_kb_1) & \cdots & \sigma^2(b_k)\end{vmatrix} \tag{9}$$

In matrix (9) we find the information on the statistical properties of the model; along the diagonal we have the estimations of dispersions of the coefficients, apart from the main diagonal we have covariance estimations.

In the equations of regression (4) the independent variables are represented under the normal form:

$$X_i^N=\frac{X_i-X_{iB}}{\Delta X_i} \tag{10}$$

X_{iB} :is the base value of the variable which is equal to the center value of the interval $\left[X_{iMIN}, X_{iMAX}\right]$. The middle of the interval is given by:

$$\Delta X_i = \frac{X_{iMAX} - X_{iMIN}}{2} \quad (11)$$

ΔX_i is the incremental variation of X_i.

The normed variables take the limit values of +1and –1. If the variables X_i defined in the interval $X_i \pm \Delta X_i$ with a probability almost equal to 1 then they are independent random variables: and the variables will be defined in the interval [-1,+1] with the same probability. They are central independent and the mathematical expectancy $E(X_i) = 0$.

The computation of the numerical data of the matrix of effects permit to represent the dependencies by simple analytical models. Each of the dependencies is approximated by a polynomial which is dependant on five variables of the type:

$F(X_1, X_2, ..., X_5, b) =$ b_{00000} + b_{10000}X[1] + b_{11000}X[1].X[2] + b_{10100}X[1].X[3] + b_{10010}X[1].X[4]+ b_{10001}X[1].X[5] + b_{20000}X[1]^2 + b_{01000}X[2] + b_{01100}X[2].X[3] + b_{01010}X[2].X[4] + b_{01001}X[2].X[5] + b_{02000}X[2]^2 + b_{00100}X[3] + b_{00110}X[3].X[4] + b_{00101}X[3].X[5] + b_{00200}X[3]^2 + b_{00010}X[4] + b_{00011}X[4].X[5] + b_{00020}X[4]^2 + b_{00001}X[5] + b_{00002}X[5]^2

5.2. The choice of the design

The choice of the design depends first on the nature of the problem (linear, non linear), secondly on financial considerations that limit the number of factors to be considered and last it depends on the precision and adequacy of the model. Bearing in mind that the major problem of the design of experiment aiming to obtain the most precise mathematical description of the object or the process is the disposition of the experimental points in the domain of factorial space under study with a minimum number of experiments. The total or partial cancellation of interactions leads to inadequate models in most of the cases and last that the design of complete factorial experiments (CFE) and the fractional factorial experiment designs (FFE) can not be used to construct second order model due to the fact that the columns in X_i^2 during all the experiments have the normed value of +1 as it is for X_0 and that the free term of the equation b_0 to will be mixed with the effects of quadratic terms. For the reasons cited above we opted for central composite orthogonal design (CCO). These designs have as a nucleus the orthogonal designs (E F C) completed with the central point $n_0 = 1$ and of $n_\alpha = 2k$ star points, arranged by pairs along the coordinate axes, and are located away from the origin by a distance α. The number of experiments required is then: $N = 2^{k-I} + 2k + 1$. Where I is the number of interactions which are substituted by new factors. The advantages of this design are the minimum variance of the output parameter and as the experiences are conducted on a computer, the CCO assures the construction of the model with minimum errors.

To transform the quadratic terms we substitute X_i^2 by X_i^{*2} in the design matrix.

$$X_i^{*2} = X_i^2 - \varphi \quad (12)$$

with

$$\varphi = \frac{2^{k-I} + 2\alpha^2}{N} \quad (13)$$

And the axial distance α is determined by the following:

$$\alpha = (\sqrt{N.2^{k-2}} - 2^{k-1})^{1/2} \quad (14)$$

The design matrix transformation allows to estimate the coefficients of the equation of regression independently of each other due to the fact that the columns X_0 and X_i^{*2} are orthogonal. In this case, the coefficients of regression are computed using (14) and their variances are found using (16).

In the expression (14) X_{0u} , X_{iu} are taken from the design matrix where $(i = \overline{1,k}\ ,\ u = \overline{1,N})$

$$\left.\begin{aligned} b_0 &= \frac{\sum_{u=1}^{N} X_{0u} Y_u}{N} & b_{ij} &= \frac{\sum_{u=1}^{N} X_{iu} X_{ju} Y_u}{2^k} \\ b_i &= \frac{\sum_{u=1}^{N} X_{iu} Y_u}{2^k + 2\alpha^2} & b_{ii} &= \frac{\sum_{u=1}^{N} X_{iu}^{*2} Y_u}{2^k(1-\varphi) + 2\alpha^2(\alpha^2 - 1)} \end{aligned}\right\} \quad (15)$$

$$\left.\begin{aligned} \sigma^2(b_0) &= \frac{\sigma^2(Y)}{N} & \sigma^2(b_{ij}) &= \frac{\sigma^2(Y)}{2^k} \\ \sigma^2(b_i) &= \frac{\sigma^2(Y)}{2^k + 2\alpha^2} & \sigma^2(b_{ii}) &= \frac{\sigma^2(Y)}{2^k(1-\varphi) + 2\alpha^2(\alpha^2 - 1)} \end{aligned}\right\} \quad (16)$$

6. COMPOSITION OF THE DESIGN MATRIX

The design matrix differs from the observed values matrix due to the fact that all elements are either equal on have values of +1 or –1 and are arranged in a special manner. For the elaboration of the design matrix we used the rule of the alternating signs (Yates' Algorithm). In the $j = 1$ column the signs are alternating by power of 2. The elements of $j = 0$ column are all equal to 1. The values of the elements of the corresponding columns to the quadratic terms are calculated by the relation (11). The number of columns of CCO matrix is 2^5 and $j = 21$ with the X_0 column, 5 columns of linear terms, 5 column of quadratic terms and 10 columns for interactions.

The number of rows is equal to the number of experiments that is $i = u = N$.

7. DETERMINATION OF THE S_I DOMAINS LIMITS OF VARIATIONS OF X_i

The regression models obtained by DOE are used in all the space of factors. It is essential to study the conditions for satisfying the restrictions (constraints) on all the points at the frontier and inside the admissible domain. If the restrictions are satisfied in all the points of the design and if this space is convex, we can conclude that the satisfaction of the restrictions of all points of the design leads to their satisfaction in all the space.

For the reasons cited above we must determine the limits of variation of the 5 variables. The determination of these limits are obtained with no problem and the variables are taken separately they are simply governed by the standards. But the difficulty arises when they are taken simultaneously especially in the case when the combinations of levels lead to conflicts.

The source of difficulty in determining the S_i domains is the test determination (combination of levels of activity) corresponding to the most unfavorable state in a sense that the risk of violating the constraints is more probable. It is from this test that we try to determine the limits of variations of the factors. The test is seeked among status where the senses of variations of certain factors are opposed to others.

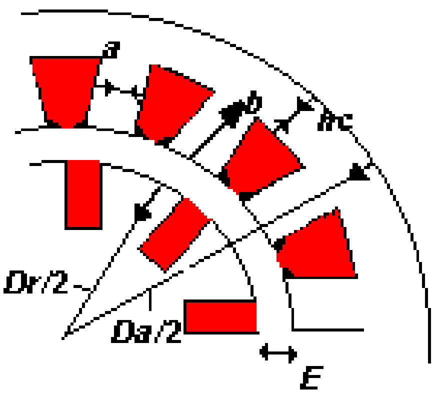

Fig. 9 Sense of variation of the geometrical dimensions

In our case, the combination sought of the levels of activities of the maintained values corresponds to the essai number (17) characterized by the levels of activity of factors and at the low level (-1), and those at high level (+1) represented by the combination (-1) (-1) (-1) (-1) (+1) which illustrates the nature of the type of conflicts which limit the S_i domains of the variation of factors. In fact for this combination the required electromagnetic force to magnetize the air-gap becomes too small but the induction in the teeth and in the cylinder head becomes too high and this in spite of the low value of the induction in the air gap that provokes on one hand the saturation of the magnetic circuit and, on the other hand the weakness of the teeth in the weakest section. In the representation of fig.9, we represent this essai by indicating the sense of variation of the geometrical dimensions dependently of the state of the factors and where "**a**" is the width and "**b**" the height of the slot "**hc**" is the height of cylinder head and the rotor diameter "$\mathbf{D_r}$".

In this considerations the limits of the variations of the factors are grouped in table II.

VCR	X1 m	X2 A/m	X3 A/mm²	X4 A/mm²	X5 mm	Limits
-1.596	0.290	30202	4.702	2.723	0.25	Lower
-1	0.305	30500	5.000	3.200	0.30	
-0.5	0.318	30750	5.250	3.600	0.34	
0	0.331	31000	5.500	4.000	0.40	Center
0.5	0.344	31250	5.750	4.400	0.45	
1	0.357	31500	6.000	4.800	0.50	
1.596	0.372	31798	6.289	5.278	0.56	Higher

Table II Symbols and units of input variables

where $X_1 = D_a ; X_2 = A; X_3 = J_s ; X_4 = J_r X_5 = E$

CRV: Centered Reduced Value

8. STANDARD PROCEDURE FOR THE COMPUTATION OF THE MOTOR

This procedure allows to determine all the quantities (geometrical, electrical and magnetic quantities) of the motor according to the standards and to study the effects of variations of the X_i quantities on the output quantities Y_i . The independent variables X_i are divided in quantitative and qualitative variables which create heterogeneity. They are considered as external factors (types of sheet, types of winding) and are maintained invariant. For all the quantitative variables having a numerical estimation we have established the major levels and intervals of variations.

The procedure is elaborated in such a way that we can vary the whole set of decision variables (quantitative) either separately or simultaneously in conformity to the type of design chosen.

In the case of the 13 kW motor with 1500 rpm; the type of winding, the forms fig. 9, the number of the type of slots and the type of the sheets used are maintained. The length of stacking and the dimensions of slots (sections) are dependant on the variations of current densities but the ratio $R_e = \boldsymbol{a}/\boldsymbol{b}$ of the width ($\boldsymbol{a}$) and the height ($\boldsymbol{b}$) are maintained constant. The diameters ratio $R_d = \boldsymbol{D_a}/\boldsymbol{D_1}$ is maintained in the limits [1.56, 1.61] where $\boldsymbol{D_a}$ is the external diameter and $\boldsymbol{D_1}$ is the boring diameter.

9. MATRIX OF EFFECTS

For the sake of economizing the space disk, we have used only one matrix of dimensions [1..26,1..48] composed of the elements of the orthogonal array, and the elements of the five columns of effects corresponding to the real parameters values of the outputs a total of 26 columns (see appendix).

10. ADEQUACY OF THE MODEL

The obtained models are statistical models, they have to be submitted to statistical tests. To verify the signification of the coefficients b_i and b_{ij}. Students' criteria is used:

$$\left|b_i\right| > \varepsilon_r = t\sigma(b) \tag{17}$$

Where t is Students' criteria value for the degree of freedom $f = N - 1$, $\sigma(b)$ is the error estimation of the coefficients b_i and b_{ij}. To determine the error $\sigma(b)$ the following formula is used:

$$\sigma^2(b_i) = \frac{\sigma^2(Y)}{\sum_{u=1}^{N} X_{iu}^2} \tag{18}$$

Where $\sigma^2(b_i)$ is the variance of reproductivity of experiments by the Y parameter. The variance $\sigma^2(Y)$ can be determined from the experiment or announced as an intrigued value of the error after a concordant investigation. The value taken to estimate the coefficients of the objective function is: $\sigma(Y) = 0{,}025\, b_0$.

After determining the value of ε_r all the coefficients such that $|b| < \varepsilon_r$ are rejected.

We can see from (18) that the variance of the coefficients $\sigma^2(b_i)$ are N times lesser than the variance $\sigma^2(Y)$. It is thus possible to obtain a satisfying precision model even in the case of a significant scattering of the experimental data.

In this case, we can increase the number of the factors and thus the number of experiments N.

This verification of the significance of the coefficients in the orthogonal designs of the second order has the following particularities:

The variances of the coefficients are determined by (18) but the values of $\sum_{u=1}^{N} X_{iu}^2$ are different for different columns. This is due to the fact that: $b_0 = b_0^{'} + \varphi \sum_{i=1}^{k} b_{ii}$, the variance $\sigma^2(b_0)$ is determined by:

$$\sigma^2(b_0) = \sigma^2(b_0^{'}) + \varphi^2 \sum_{i=1}^{k} \left[\sigma^2(b_{ii})\right] \tag{19}$$

11. CHECK IN OF THE MODEL ADEQUACY

After eliminating the non significant terms, the obtained equation have to be submitted to the tests of adequations (proximity of the object and the model). Ficher's criteria usually used for the comparison of variances is applied:

$$\left.\begin{aligned} & F = \frac{\sigma_{ad}^2(Y)}{\sigma_{rep}^2(Y)} \\ & \sigma_{ad}^2(Y) = \frac{\sum_{u=1}^{N}(Y_u - \hat{Y}_u)^2}{f_1} \qquad \sigma_{rep}^2(Y) = \frac{\sum_{q=1}^{l}(Y_q - \hat{Y}_q)^2}{f_2} \end{aligned}\right\} \tag{20}$$

- $f_1 = N - q$ and $f_2 = f - f_1 = q - 1$, where:

$\sigma_{ad}^2(Y)$: is the variance of adequation;

$\sigma_{rep}^2(Y)$: is the variance of reproducivity;

f : the number of degree of freedom;

$\hat{Y}_u$: Computed value starting from the regression equation

q : the number of the approximation polynomial terms.

For the verification of the model adequancy the computed values are compared to the data values by Ficher's table for the desired level of significance. The calculated value is:

$$F_{cal} = \frac{\sigma_{ad}^2(Y)}{\sigma_{rep}^2(Y)} \tag{21}$$

The adopted level of significance for verifying the models adequancy is: $p = 0{,}05$. if $F_{cal} < F_F$ we can consider the model as adequate with the probability $1 - p$.

The compared empirical variances are considered as estimations of the same collective variance. To verify the significance of the coefficients and the adequation, it is necessary to determine the variance of reproducivity of parallel experiments $\sigma_{rep}^2(Y)$. Conformely to (9) and in the case where the variables are disposed according to a normal law, the value of $\sigma_{rep}^2(Y)$ is determined by the following expression:

$$\sigma_{rep}^2(Y) = 0{,}11 \sum_{i=1}^{k} b_i^2 \tag{22}$$

For the statistical analysis of the obtained regression equations the variance of reproducivity of experiments can also be estimated by the minimum value of the coefficient of the significantly influent factor, then:

$$|b_i| > \frac{t\sigma_{rep}(Y)}{\sqrt{N}} \tag{23}$$

and the mean quadratic error of reproducivity is

$$\sigma_{rep}(Y) = \frac{|b_i|\sqrt{N}}{t} \tag{24}$$

The adequation according to Ficher's criteria is verified if:

$$F_F = \frac{\sigma_{ad}^2}{\sigma_{rep}^2} > F_{Cal} \tag{25}$$

12. INTERPRETATION OF THE RESULTS

Due to the fact that CCO has CFE as nucleus, all the interactions are used and then $I = 0$ for $k = 5$ the number of experiments is $N = 32 + 10 + 1 = 43$ the axial distance α=1.596, the transformation coefficient is φ= 0.862.

To test the significance of the coefficient we impose to the reproducivity error to be less than 2.5% (for the cost); that gives $\sigma(Y) = 63.14$. The value of b_{min} for a degree of freedom $f = 42$ the level of significance p=5% and the value of Student's criteria equals to 1.75; we have $b_{min} = 19.258$. The value of Ficher's criteria for a degree of freedom $f_1 = 30$ and $f_2 = 12$ is $F_F = 2.2$. Compared to the calculated value $F_{Cal} = 1.3889$ we deduce that the model is adequate. The values of the coefficients of the regression equations of the models are given in table III, where it can be seen that for the ten (10) interactions used only the $X_1 X_4$ and $X_3 X_5$ interactions are no significant and they can be substituted by new factors. or reduce the number of experiments.

Coefficients	Cost D.a	output	T^O C	Cos(φ)	Kmax
b $_{00000}$	2525.72	0.8945	45.54	0.88744	1.99895
b $_{10000}$	247.62	0.002	43.97		
b $_{11000}$	38.21				0.02838
b $_{10100}$	25.63		0.27		0.02438
b $_{10010}$					
b $_{10001}$		0.0011	-0.23		0.02619
b $_{20000}$		0.0015	0.43		0.03329
b $_{01000}$	27.95	-0.0002	0.5		
b $_{01100}$			-0.18		
b $_{01010}$				0.00438	
b $_{01001}$					0.01850
b $_{02000}$	85.67	0.0029	0.45	-0.0041	0.05959
b $_{00100}$	-23.47	-0.0027	4.04	0.00423	
b $_{00110}$	19.66			0.00313	
b $_{00101}$					
b $_{00200}$	26.91	0.0025	0.52		
b $_{00010}$	95.55	-0.0025		0.00554	0.06066
b $_{00011}$	-66.51				-0.02400
b $_{00020}$	-44.29		0.34	-0.0062	-0.03817
b $_{00001}$	-195.61	-0.0021	0.57		
b $_{00002}$			0.23		-0.04562

Table III Regression coefficients

σ$_{\text{reference mark}}$ (y)	63.14	0.0027	0.5	0.01	0.05000
F_{Cal}	1.3889	1.6375	1.632	1.1791	1.40440
F_F	2.2	2.2	2.2	2.2	2.2
b $_{min}$	19.258	0.0008	0.151	0.004	0.01520

Table IV Statistical tests values able 4 optimal Alternative

External Diameter D $_{has}$ [mm]	318	Height of the stator cylinder head [mm]	47,5
Internal diameter D $_1$ [mm]	167	A rotor number of Slots Z2	46
Air-gap E [mm]	0.35	Height of the rotor Slot [mm]	22,9445
Induction in the air-gap [Tesla]	0.727	Width of the rotor Slots [mm]	3.3637
Linear density A [A/m]	30750	Length of the armature [mm]	150,5
Density of current stator Js [A/mm²]	5.25	Cost [D.A]	2576.17
Density of current rotor Jr [A/mm²]	3.6	Efficiency η	0.904
Density of current of the rings [A/mm²]	2.6	Power-factor Cos (φ)	0.902
A number of coils W	108	Capacity of overload Kmax	2.059
A number of stator Slots Z1	36	Torques ratio C $_D$ / C $_N$	1.315
Average width of the stator tooth [mm]	8,4	Currents ratio I $_D$ / I $_N$	5.126
Height of the stator tooth [mm]	28,13	Temperature of the stator T^O (C)	44.91

Table V Optimal alternative

The major dimensions and performances of the motor corresponding to the optimal alternative of abscissa (-0.5) are summarised in table IV. We observe that the efficiency is very high just as the power-factor which enables us to affirm that the performances were improved

13. CONCLUSION

The search of the optimal alternative by the traditional techniques of optimization such as the Lagrange's method and the methods of the gradients require the repetition from the beginning of all the computing process long and complicated. This characteristic is due to the fact that information on the properties and on the characteristics obtained by these methods are neither accumulated nor generalized. It is possible to avoid this defect, by using DOE which accumulates and stores the dependencies and explicitly reflects them between the technico-economical parameters necessary to the search of the optimal alternative.

The originality of this approach consists in the strategy adopted in the study and in the representation which allows to avoid the use of the techniques of complex optimization. The optimal alternative can be identified quite simply. The effectiveness of DOE for the problem of multi-objectives optimization is proved

For the case under study the method is used to minimize the cost of active materials using DOE, it minimizes the number of experiments and gives the most influencing parameters on the design. It also allows to see the other performance parameters such as efficiency, power factor, torque ratio, … Thus the method is multi-objective and can be used for any application for design and construction.

14. SPECIFICATIONS

The specifications consist in minimizing the cost of active materials of an asynchronous squirrel-cage motor of 13kW 1500rpm 220/380 Volts, ambient temperature 40^0 (C) such that:

1. Efficiency $\eta = H_1(X_1) \geq 0.88$
2. Power factor $Cos(\varphi) = H_1(X_i) \geq 0.8$
3. Overload capacity $K_{max} = G_1(X_i) > 2$
4. Temperature T(^{0}C) $T = G_2(X_i) < 60$
5. Current ratio $I_d / I_n = G_3(X_i) < 6$

15. APPENDIX

N	X0	X1	X2	X3	X4	X5	X1X2	…	X4X5
1	1	-1	-1	-1	-1	-1	1	…	1
2	1	1	-1	-1	-1	-1	-1	…	1
3	1	-1	1	-1	-1	-1	-1	…	1
4	1	1	1	-1	-1	-1	1	…	1
5	1	-1	-1	1	-1	-1	1	…	1
6	1	1	-1	1	-1	-1	-1	…	1
7	1	-1	1	1	-1	-1	-1	…	1
8	1	1	1	1	-1	-1	1	…	1
9	1	-1	-1	-1	1	-1	1	…	-1
10	1	1	-1	-1	1	-1	-1	…	-1
11	1	-1	1	-1	1	-1	-1	…	-1
12	1	1	1	-1	1	-1	1	…	-1
13	1	-1	-1	1	1	-1	1	…	-1
14	1	1	-1	1	1	-1	-1	…	-1
15	1	-1	1	1	1	-1	-1	…	-1
16	1	1	1	1	1	-1	1	…	-1
17	1	-1	-1	-1	-1	1	1	…	-1
18	1	1	-1	-1	-1	1	-1	…	-1
19	1	-1	1	-1	-1	1	-1	…	-1
20	1	1	1	-1	-1	1	1	…	-1
21	1	-1	-1	1	-1	1	1	…	-1
22	1	1	-1	1	-1	1	-1	…	-1
23	1	-1	1	1	-1	1	-1	…	-1
24	1	1	1	1	-1	1	1	…	-1
25	1	-1	-1	-1	1	1	1	…	1
26	1	1	-1	-1	1	1	-1	…	1
27	1	-1	1	-1	1	1	-1	…	1
28	1	1	1	-1	1	1	1	…	1
29	1	-1	-1	1	1	1	1	…	1
30	1	1	-1	1	1	1	-1	…	1
31	1	-1	1	1	1	1	-1	…	1
32	1	1	1	1	1	1	1	…	1
33	1	-1.596	0	0	0	0	0	…	0
34	1	1.596	0	0	0	0	0	…	0
35	1	0	-1.596	0	0	0	0	…	0
36	1	0	1.596	0	0	0	0	…	0
37	1	0	0	-1.596	0	0	0	…	0
38	1	0	0	1.596	0	0	0	…	0
39	1	0	0	0	-1.596	0	0	…	0
40	1	0	0	0	1.596	0	0	…	0
41	1	0	0	0	0	-1.596	0	…	0
42	1	0	0	0	0	1.596	0	…	0
43	1	0	0	0	0	0	0	…	0

Table VI-A Matrix of design and effects

N	$X1^{*2}$	…	$X5^{*2}$	Cout	η	Cos(φ)	T (^{0}C)	Kmax
1	0.138	…	0.138	2412.65	0.902	0.881	45.54	1.991
2	0.138	…	0.138	2819.43	0.903	0.89	43.97	1.893
3	0.138	…	0.138	2374.25	0.9	0.874	46.06	1.879
4	0.138	…	0.138	2867.89	0.902	0.872	45.3	1.894
5	0.138	…	0.138	2316.75	0.897	0.88	53.18	1.978
6	0.138	…	0.138	2719.46	0.898	0.89	51.62	1.881
7	0.138	…	0.138	2283.43	0.895	0.875	53.86	1.865
8	0.138	…	0.138	2771.57	0.896	0.872	53.23	1.882
9	0.138	…	0.138	2760.74	0.896	0.895	46.24	2.195
10	0.138	…	0.138	3010.02	0.898	0.815	43.48	1.982
11	0.138	…	0.138	2606.92	0.894	0.89	46.81	2.084
12	0.138	…	0.138	3069.7	0.898	0.913	44.24	1.974
13	0.138	…	0.138	2673.9	0.89	0.886	53.81	2.18
14	0.138	…	0.138	3174.55	0.892	0.909	52.21	2.06
15	0.138	…	0.138	2608.98	0.89	0.9	53.51	2.036
16	0.138	…	0.138	3243.24	0.89	0.898	53.97	2.052
17	0.138	…	0.138	2155.64	0.898	0.881	46.99	2.049
18	0.138	…	0.138	2422.24	0.897	0.89	44.48	1.851
19	0.138	…	0.138	2076.39	0.892	0.857	48.7	1.963
20	0.138	…	0.138	2670.39	0.903	0.88	45.47	2.042
21	0.138	…	0.138	1996.49	0.888	0.881	55.38	1.927
22	0.138	…	0.138	2525.82	0.896	0.888	53.13	2.029
23	0.138	…	0.138	2021.83	0.89	0.882	55.39	1.928
24	0.138	…	0.138	2575.13	0.897	0.883	53.77	2.025
25	0.138	…	0.138	2259.74	0.894	0.893	46.11	2.151
26	0.138	…	0.138	2618.45	0.894	0.897	44.82	2.049
27	0.138	…	0.138	2031.5	0.883	0.847	49.6	1.984
28	0.138	…	0.138	2645.75	0.895	0.895	45.34	2.05
29	0.138	…	0.138	1971.05	0.881	0.888	55.19	1.937
30	0.138	…	0.138	2501.84	0.889	0.898	52.8	2.038
31	0.138	…	0.138	2121.47	0.887	0.895	54.29	2.017

32	0.138	…	0.138	2748.57	0.891	0.897	53.43	2.12
33	1.685	…	-0.862	2036.97	0.891	0.894	50.57	1.925
34	1.685	…	-0.862	2959.93	0.899	0.905	47.79	2.204
35	-0.862	…	-0.862	2466.67	0.898	0.906	47.77	2.031
36	-0.862	…	-0.862	2879.27	0.899	0.881	50.7	2.225
37	-0.862	…	-0.862	2632.24	0.903	0.903	41.75	2.031
38	-0.862	…	-0.862	2429.98	0.892	0.906	57.05	2.001
39	-0.862	…	-0.862	2200.34	0.892	0.868	49.6	1.761
40	-0.862	…	-0.862	2518.09	0.891	0.909	48.32	2.023
41	-0.862	…	1.685	2729.25	0.897	0.903	47.62	1.864
42	-0.862	…	1.685	2174.77	0.889	0.888	49.75	1.884
43	-0.862	…	-0.862	2522.65	0.898	0.905	48.41	2.02

Table VI-B Matrix of design and effects

REFERENCES

[1] LINDSAY S.F & BARTON T.H, Parameter identification for squirrel cage induction machines I.E.E.E Trans Power Appar sys 1973 flight 4 pages 1287-1291

[2] ERAJSKAR G, BHATTACHARYYA M & MAHENDRA SN, Computer aided design of three phase squirrel cage, induction motor Technical design, hybrid proceess and optimization J.Instit Engrs (India) Electr.Engng Div India 1974 Flight 55 NR 2 pages 2-50

[3] BHARADWAY.DG, VENKATESAN K & SEXEMA RB, Induction motor design optimization using constrained, Rosenbrock method (Hill algorithm), GBR, 1979 Flight 6 NR

[4] FIACCO AV and Mc CORMIK GP,Nonlinear programing SUMT, Mgmt Sci 10 19

[5] Fulton N.N., Slater R.D. and Wood W.S., Design optimization of small 3-phase induction motors, PROC IEE. Vol 123,No 2 February 1976 ROGER PHAN-TANLUU DIDIER MATHIEU DANIEL FENEUILLE Experimental design methodology study of surfaces of reactionnelle answer laboratoirede prospective and analysis of information Marseilles 1983

[6] .Box G.E.P and N.R. To drape, Empirical model-building and response surfaces, John Wiley & Sounds Inc New York NY 1987

[7] RAMARATNAM R & DESAI BG & SUBRA RAO V, A Comparative study of technical minimization for optimization of induction motor I.E.E.E Winter power New York Meeting 1977 PAPER A 77 pages 259 - 263

[8] RAMARATNAM R & DESAI BG, optimization of polyphase induction motor design anonlinear programming approch I.E.E.E Trans Power Appar Sys 1970 step 90 pages 570-578

[9] R.w Menzies & G.n Neal, Optimisation Programs Broad For Induction Motor Design I.E.E Proceeding B Yellow 1975 flight 122 6 pages 63

[10] POWELL M.J.D, A method nonlinear constraints in minimization problems, Paper presented At the conference one optimization University College of North Staffordshire 1968,

[11] N. MOKHTARI M.RAHLI Y. RAMDANI DOE & technique of optimization of the electric motor, I.C ESSA Blida Algeria 99

[12] Mr. Saharaian and S. Kodivalam, Tuning PID controllers using error-integralcriteria and numerical optimization, Symposium one multidisci- plinary analysis and optimization 1996 AIAA-96-

[13] S. Kodyamam, L Swensonand B.Stehlin, Multidisciplinary design optimization with object-oriented product modeling Proceedings optimization in industry conference, March 1997

[14] G.Tagauchi and S. Konishi, Orthogonal arrays and linear graphs, American to beg institute, Dearborn, MID 1987

[15] SRINIVASS KODIYAM JIAN KNOWN FLAX AND BRET A. WUJEK DOE based response surface models for design optimization, Engineous sofware Inc Published by the Americain institute of aeronautics and astronautics, Inc 1998

BIOGRAPHY

Naimi Mokhtari was born in El Bayadh, Algeria, on 1958. He graduated from the Algeria General Teaching College, and studied at the University of Oran. His employment experience included the professor in electrical engineering at the institute of technology of education Oran Algeria, Algerian Textile Company, Assistant professor at University of Sidi Bel Abbes Algeria, Founder member of laboratory IRECOM. His special fields of interest included optimization of process using deigns of experiments and data mining.

Abdelkader Zeblah was born in 1965 in Sfisef, Sidi-Bel abbes, Algeria. He received his BS degree in electrical engi- neering from the Electrical Engineering Institute of The Uni- versity of Sidi Belabbes (Algeria) in 1990, the MS degree from the Electrical Engineering Institute of The University of Sci- ences and Technology of Oran (USTO) in 1993, and the PhD degree from the Electrical Engineering Institute of The University of Sciences and Technology of Oran (USTO) in 2001. He is currently Professor of electrical engineering at The University of Sidi Belabbes (Algeria). His research interests include operations, planning and economics of electric energy systems, as well as optimization theory and its applications.

Abdelkader Lousdad born in 1955 at Sidi Bel Abbes. Graduated from the University of Cincinnati USA in 1979 in mechanical engineering. Hold a Master in Industrial Management of Enterprise 1989 and Magister 1992 in structure Robotics from the University of Sidi Bel Abbes. Former mechanical and industry (production, assembly, maintenance quality control product design and manufacturing. Has been central technical and research coordinator, then a supervisor head of industrial engineering design unit. Research interest: Research focus is on applying system concepts to engineering sciences and technologies.

Yamani Massim born in 1959 at M'sirda. Graduated from the University of Aston in Birmingham Uk in 1983 in mechanical engineering. Obtained a Master in Safety Engineering in 1984 from the same university and PhD in 2005 From the University of Sidi Bel Abbes (Algeria). Lecturer of mechanic's since 1985 at department of mechanical engineering university of Sidi Bel Abbes . Research focus is on optimization of industrial system.