FUZZY APPROACHES TO DISTRIBUTION ENERGY LOSSES CALCULATION

Dragan TASIĆ, Miodrag STOJANOVIĆ
Department of Energetics, Faculty of Electrical Engineering, University of Niš
Beogradska 14, 18000 Niš, Serbia and Montenegro, tel.: +381 18 529 135, E-mail: dtasic@elfak.ni.ac.yu

SUMMARY

Two approaches to calculating distribution energy losses are developed in this paper. The first one is based on the fuzzy load flow, and the other one uses fuzzy clustering technique. The attention at the first approach is devoted to forming the fuzzy numbers that represent loads. Data accessible from the measurements in corresponding substations are considered in this process. Using formed fuzzy loads one fuzzy load flow calculation is made. Results of calculation are fuzzy power losses. Defuzzification gives the deterministic value of average power losses that multiplying with the number of hours for analyzed period gives energy losses. For the second approach, the range of the coefficient that defines fuzziness of clustering is determined, as well as an optimal number of clusters. Analyses shows that the best results are obtained for fuzziness coefficient on the range of 1.1-2, and the number of clusters up to 20.

Keywords: *losses, electrical energy, fuzzy, clustering, load flow*

1. INTRODUCTION

Power and energy losses are inevitable consequence of energy transmission and distribution from generation to consumer points. The total losses sometimes make ten or more percents of delivered energy. Therefore, it is important to have the right estimate of losses, as well as to find ways for their reduction. Basic difficulties in solving these tasks are: identification of technical and non-technical losses, determination of the structure of losses (distribution of losses throughout the network elements), location of critical elements from the aspect of losses, and selection of optimal methods for losses reduction. The importance of technical losses becomes even higher for distribution utilities in deregulated environment since the non-technical losses will become out of concern (retail companies will take care of them).

Identification of load curves for each particular element is needed for exact calculation of technical losses. That is not possible since appropriate meters exist only at some locations in the network. This is one of causes that many different approaches for distribution losses assessment are developed. All these approaches can be classified in two basic groups: deterministic and probabilistic [1, 10 and 11]. Depending on factors, chosen to be the most influential, in each of these two main groups we may distinguish some subgroups. The variety of the methods is due to chosen factors, but at the same time illustrates the complexity of the electric energy loss calculations as well as impossibility to find unique approach.

In a large variety of deterministic methods, the method based on the relationship between load and loss factors has been widely used. Methods based on equivalent impedance, average current and average square current, as well as regression method are also frequently used [10]. These methods can give results that significantly differ from real values in some cases. Moreover, application of these methods to finding structure of losses is limited. That is why we should improve existed methods as well as develop new ones.

As a result of the fact that loads are not exact known, the fuzzy load flow method [7] is developed. Starting from this method, an approach to calculating energy losses is developed in this paper. The attention is devoted to forming the fuzzy numbers that represent node loads.

Distribution networks are extended over wide areas and consisted of large number of elements. Because of that, last years has appeared method for electrical energy losses calculation based on clustering technique. In this paper, a method for energy losses calculation, based on fuzzy clustering technique, is developed.

2. APPROACH BASED ON THE FUZZY LOAD FLOW

In distribution networks measurements are usually made only for industry consumers, and on the HV/MV substations. Therefore loads (powers) for many load nodes are not exact known, but they can be estimated in some way.

Since the loads (powers) are not strictly known, it is appropriate to consider them as fuzzy numbers. Power limits (min and max), for each load node, can be assessed on the basses of experience. Beside loads, we can regard voltage of distribution root node as fuzzy number. If fuzzy numbers that represent loads as well as voltage of root node are determined, we can make fuzzy load flow calculation [7]. Procedure of calculation is based on the algorithm presented in [8] according to fuzzy arithmetic lows. The usual algebraic operations can readily be extended to fuzzy sets using the Extension Principle formulated by Zadeh. For any α -interval of confidence is:

$$C^{(\alpha)} = A^{(\alpha)}(+)\,B^{(\alpha)} = [\,A_1^{(\alpha)} + B_1^{(\alpha)}\,;A_2^{(\alpha)} + B_2^{(\alpha)}\,] \quad (1)$$

$$C^{(\alpha)} = A^{(\alpha)}(-)\,B^{(\alpha)} = [\,A_1^{(\alpha)} - B_2^{(\alpha)}\,;A_2^{(\alpha)} - B_1^{(\alpha)}\,] \quad (2)$$

ISSN 1335-8243

$$C^{(\alpha)} = A^{(\alpha)}(\cdot)B^{(\alpha)} = \\ =[\min[A_1^{(\alpha)} \cdot B_1^{(\alpha)}, A_2^{(\alpha)} \cdot B_1^{(\alpha)}, A_1^{(\alpha)} \cdot B_2^{(\alpha)}, A_2^{(\alpha)} \cdot B_2^{(\alpha)}] \\ \max[A_1^{(\alpha)} \cdot B_1^{(\alpha)}, A_2^{(\alpha)} \cdot B_1^{(\alpha)}, A_1^{(\alpha)} \cdot B_2^{(\alpha)}, A_2^{(\alpha)} \cdot B_2^{(\alpha)}]] \tag{3}$$

$$C^{(\alpha)} = A^{(\alpha)}(/)B^{(\alpha)} = \\ =[\min[A_1^{(\alpha)} / B_1^{(\alpha)}, A_2^{(\alpha)} / B_1^{(\alpha)}, A_1^{(\alpha)} / B_2^{(\alpha)}, A_2^{(\alpha)} / B_2^{(\alpha)}] \\ \max[A_1^{(\alpha)} / B_1^{(\alpha)}, A_2^{(\alpha)} / B_1^{(\alpha)}, A_1^{(\alpha)} / B_2^{(\alpha)}, A_2^{(\alpha)} / B_2^{(\alpha)}]] \tag{4}$$

Fuzzy load flow calculations are usually made using triangular or trapezoidal fuzzy numbers. It should noticed, that addition and subtraction of these fuzzy numbers do not deform membership functions. If triangular (trapezoidal) fuzzy numbers are multiplied or divided, membership functions of result is not triangular (trapezoid) fuzzy number.

If we want to apply fuzzy load flow for distribution energy losses calculation basic problem is forming of fuzzy numbers that represent loads. Therefore, a new method to forming fuzzy numbers that represent loads is proposed in this paper. The method gives satisfactory results for energy losses, as we will show in test example.

The estimation process considers assumption of exact knowing the following data:

- current curve of the first feeder section,
- peak power of each load node,
- participations of load categories in each node,
- typical load patterns of different load categories.

Current curve of the first feeder section is usually known on the bases of meters made at substation. Peak powers of load nodes we can assess on the bases of rated power of distribution transformers and service experience. A distribution transformer supplies consumers that belong to different load categories. Because of that, it is eligible to know load patterns of different load categories, as well as participations of load categories in load nodes. These data we can obtain by meters on proper locations and analyzing historical service data.

On the bases of data mentioned above, fuzzy numbers that represent different load categories can be formed. For this reason, at the first annual current curves for each load category are estimated. Current of all consumers that belong to x load category for hour t of year (correspond to hour j of day) we can estimate as:

$$I_x(t) = k(j)\, I_{fs}(t)\, p_x^d(j) \sum_{i\in\beta} k_{x_i} k_i I_{ri} \quad x \in \alpha_L \ , \tag{5}$$

where:

x - load category,

$I_{fs}(t)$ - current of first feeder section at hour t,

$p_x^d(j)$ - relative power of x load category at hour j given from a typical hourly load pattern,

$k_i I_{r_i}$ - assumed maximal current of distribution transformer i.

k_{x_i} - participation of x load category in node i,

β - set of load nodes,

α_L - set of load categories,

$p_x^d(j) k_{x_i} k_i I_{ri}$ - assumed current of x load category consumers in node i at the hour j of the day when annual peak loading appears.

The coefficient $k(j)$ in (5) is calibration coefficient that can be calculated using relation [5, 9]:

$$k(j) = \frac{1}{\sum_{i\in\beta} k_i\ I_{ri} \sum_{x\in\alpha_L} k_{x_i} p_x^d(j)} \ . \tag{6}$$

Introducing this coefficient, it is satisfied request $\sum_{x\in\alpha_L} I_x(t) = I_{fs}(t)$. The current curves are then normalized using equation:

$$I'_x(t) = \frac{I_x(t)}{\max_t I_x(t)} \qquad x \in \alpha_L \ . \tag{7}$$

Based on the normalized curves, probability distribution of current of each load category is obtained determining number of hour for each 1% segment of loading. These distributions are converted to fuzzy membership functions according to a possibility-probability consistency principle [2]. Membership functions, obtained on this way, have complex forms, but they can usually approximate by some simple curves. Fuzzy numbers consisting of three lines are used for load categories in test example presented in this paper.

Current of load node i, as fuzzy number, can be estimated by equation:

$$\tilde{I}_i = k_i\, I_{ri} \sum_{x\in\alpha_L} \tilde{I}_x\, k_{x_i} \ . \tag{8}$$

Using estimated fuzzy currents of load nodes fuzzy load flow calculation is made. Results of calculation are node voltages and power/current flows as fuzzy numbers. Using the calculated current values, power losses as fuzzy numbers can be obtained. Defuzzification gives the deterministic value of power losses that multiplied with number of hours for analyzed period (8760 i.e. 8784 for leap year) gives energy losses. There are several defuzzyfication strategies, but authors suggest the bisector method [3].

Accuracy of presented method is analyzed on the test example, comparing its results with ones obtained by deterministic approach. In this case, current of load node i, for hour t of year can be estimated as:

$$I_i(t) = k(j)\, I_{fs}(t)\, k_i\, I_{ri} \sum_{\alpha_L} k_{x_i}\, p_x^d(j) \ . \qquad (8)$$

Using these currents, load flow calculation [8] is made for each hour of year (8760 or 8784 for leap year). Annual energy losses are determined as sum of calculated power losses.

3. APPROACH BASED ON FUZZY CLUSTERING TECHNIQUE

Clustering is one of methods for analyzing and processing large and not well-known amount of data. This is the method of classifying the data set X into subsets, clusters, based on a defined similarity measure [5, 6]. On this way, a set of characteristic states that describe analyzed problem can be generated.

In classical clustering analysis, these classes are required to form a partition of X such that degree of association is strong for data within blocks of the partition. When the requirement of a crisp partition of X is replaced with a weaker requirement of a fuzzy partition or a fuzzy pseudopartition of X (Appendix 1), we refer to the emerging problem area as fuzzy clustering. Fuzzy pseudopartitions are often called fuzzy c partition, where c designates the number of fuzzy classes in the partition.

There are two basic methods of fuzzy clustering. One of them, based on fuzzy c partitions, is called a fuzzy c means clustering method [3]. Mentioned algorithm is given in appendix 2. The algorithm starts from the assumption that desired number of clusters c is given and, in addition, a real number m, and a small positive number ε, serving as a stopping criterion, are chosen.

For purpose of loss calculations clustering process is performed on the state vector consisting of active and reactive powers of load nodes and root node voltage for each hour during the year. If estimations of network states are made on the bases of data mentioned above, instead of node powers (active and reactive) we can use node currents estimated using (8). As result of clustering process we obtain c clusters, i.e. c characteristic states of the network. After that, fuzzy load flow calculations of characteristic states of network are made [8]. Results of calculations are node voltages, power flows, and power losses in every single element of network, as well as the total power losses for cluster centers. Now we assume that loadings are constant during the set of hours that belong to the cluster. Multiplying power losses of cluster center with the number of hours that belong to the cluster, energy losses of cluster are obtained. Total energy losses are calculated as sum of energy losses of clusters.

4. TEST EXAMPLE

The methods, presented above, are used for calculation of annual energy losses of a few test networks. One of used test networks is shown in Fig. 1. This is real 10 kV suburban distribution network consisting of 28 overhead lines, 5 cable lines (dashed line), and 21 distribution substations 10/0.4 kV/kV. Fig. 1 contains the parameters required for calculations.

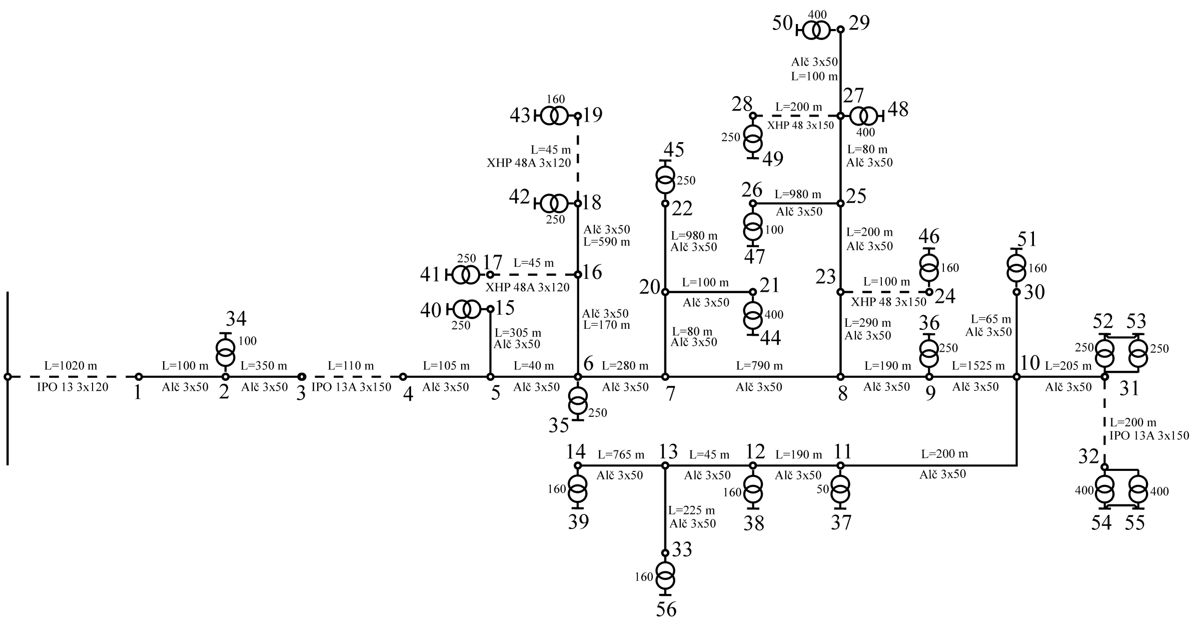

Fig. 1 Test network

The consumers were classified into three categories denoted as *A*, *B* and *C* load category. Fig. 2 shows normalized hourly load patterns of different load categories [4], while data about annual peak powers of load nodes and participations of load categories in each load node are shown in Table 1.

Fuzzy numbers that represent loading for each load category, shown on the figures 5-7, were formed on the basis of typical hourly load patterns (Fig. 2) and measured current curve of the first feeder section (Fig. 3) or duration current curve (Fig. 4). Membership functions, of these fuzzy numbers, consisting of three lines were obtained from probability functions using least square method. Using these fuzzy numbers and the data given in the table 1, fuzzy loadings of load nodes were determined. Then, fuzzy load flow were made, and fuzzy power losses were calculated.

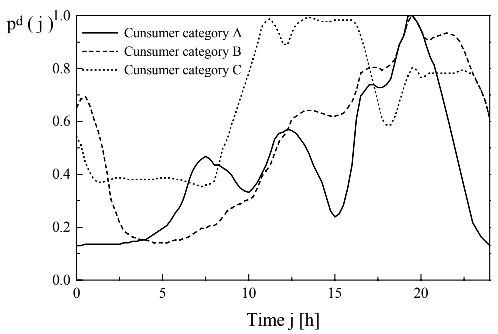

Fig. 2 Hourly load patterns for different load categories

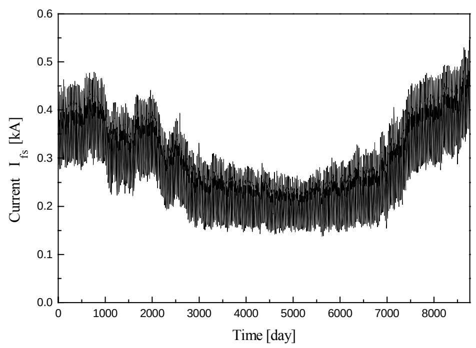

Fig. 3 Current curve of the first feeder section

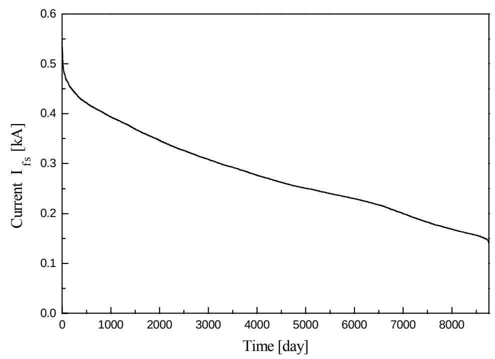

Fig. 4 Duration current curve of the first feeder section

Defuzzyfication of calculated fuzzy power losses gave us average power losses. Defuzzyfication were made using bisector method. Annual energy losses were calculated multiplying average power losses with the number of hours (8760 or 8784).

Node number	A (%)	B (%)	C (%)	S_{max} [MVAr]	cos φ
34	70	30	0	0.090	0.96
35	60	20	20	0.220	0.96
36	100	0	0	0.240	0.95
37	20	20	60	0.020	0.97
38	20	0	80	0.170	0.98
39	0	20	80	0.170	0.96
40	20	50	30	0.180	0.94
41	30	70	0	0.180	0.93
42	80	20	0	0.200	0.95
43	0	80	20	0.200	0.98
44	60	40	0	0.300	0.96
45	90	0	10	0.180	0.95
46	0	90	10	0.170	0.99
47	10	20	70	0.05	0.99
48	10	10	80	0.350	0.96
49	0	20	80	0.230	0.97
50	20	30	50	0.380	0.95
51	20	20	60	0.160	0.96
52	20	0	80	0.270	0.94
53	20	0	80	0.270	0.94
54	20	20	60	0.380	0.97
55	20	20	60	0.380	0.97
56	10	50	40	0.160	0.96

Tab. 1 Peak powers and the participation of each load category

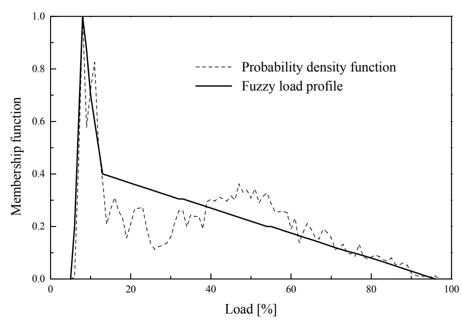

Fig. 5 Fuzzy load profile of A load category

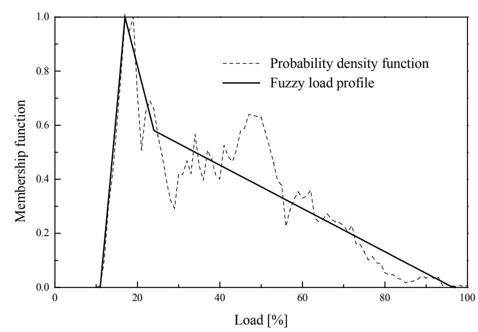

Fig. 6 Fuzzy load profile of B load category

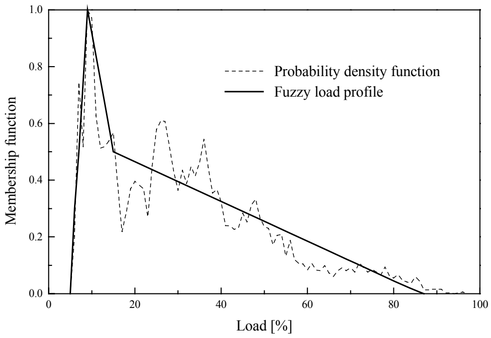

Fig. 7 Fuzzy load profile of C load category

When electrical energy losses were calculated using fuzzy clustering, we changed number of clusters *c* as well as the value of coefficient *m* that defines fuzziness of clustering.

In order to show accuracy of presented methods, loading of load nodes, for each hour, are estimated using deterministic approach. After that, annual energy losses are determined as sum of power losses, obtained from load flow calculations for each hour during the year. Results obtained on this way we regard as accurate. Deterministic, fuzzy, and fuzzy clustering calculations are based on the same estimation method. Therefore, differences in results are only consequence of using fuzzy, i.e. fuzzy clustering approach. Real accuracy of presented methods depends obviously on the accuracy of estimation procedure.

Results of deterministic, fuzzy load flow, and fuzzy clustering calculations are shown on table 2. The table, beside total energy losses (real and reactive), shows line losses apart of transformer losses. Additionally, losses of some chosen elements are shown.

The reference values of losses (exact losses) are ones given in the row “estimation”. Comparing the results, following statements can be established.

			Total losses		Lines losses		Transformers losses		Line 4-5 losses		Line 9-10 losses		Transformer 30-51 losses	
			MWh	MVArh	MWh	MVArh	MWh	MVArh	MWh	MVArh	MWh	MVArh	MWh	MVArh
Estimation			780.2	709.2	530.9	321	249.3	388.2	30.98	18.5	75.4	45.02	9.795	15.3
Fuzzy approach			794.1	735.3	560.7	339.1	255	396.2	33.2	19.8	70.1	41.9	7.8	10.3
Fuzzy clustering	m=1.1	1	655.7	570.7	434.2	262.5	221.5	308.2	25.26	15.08	62.51	37.32	8.766	12.69
		2	744	665.1	504.5	305	239.4	360	29.46	17.59	71.81	42.88	9.485	14.51
		3	763.4	686	520	314.4	243.4	371.6	30.37	18.13	73.93	44.14	9.643	14.91
		5	769.4	693.9	524	316.8	245.3	377	30.61	18.27	74.36	44.4	9.690	15.03
		10	774.5	701	527.3	318.8	247.2	382.2	30.79	18.38	74.82	44.67	9.720	15.11
		20	777.1	705.1	528.9	319.8	248.2	385.3	30.87	18.43	75.1	44.83	9.766	15.22
	m=1.25	1	655.7	570.7	434.2	262.5	221.5	308.2	25.26	15.08	62.51	37.32	8.766	12.69
		2	743.1	665	504.5	305.1	239.4	359.9	29.46	17.58	71.82	42.89	9.478	14.48
		3	764.4	687	520.8	314.9	243.6	372.1	30.41	18.16	74.03	44.2	9.643	14.91
		5	769.1	693.6	523.8	316.7	245.3	376.9	30.58	18.26	74.36	44.4	9.682	15.01
		10	773.3	700.4	526.2	318.1	247.2	382.2	30.72	18.34	74.7	44.6	9.724	15.12
		20	777.5	705.5	529.2	320	248.3	385.4	30.88	18.44	75.16	44.87	9.763	15.22
	m=1.5	1	655.7	570.7	434.2	262.5	221.5	308.2	25.26	15.08	62.51	37.32	8.766	12.69
		2	744	665	504.6	305.1	239.4	359.9	29.45	17.58	71.84	42.89	9.469	14.47
		3	765	687.4	521.3	315.2	243.7	372.2	30.43	18.17	74.09	44.23	9.626	14.87
		5	769.6	693.8	524.3	317	245.3	376.8	30.59	18.26	74.49	44.47	9.675	14.99
		10	771.3	698	524.6	317.2	246.7	380.9	30.62	18.28	74.51	44.49	9.710	15.08
		20	776.6	704	528.7	319.7	247.9	384.4	30.84	18.41	75.09	44.83	9.739	15.16
	m=2	1	655.7	570.7	434.2	262.5	221.5	308.2	25.26	15.08	62.51	37.32	8.766	12.69
		2	721.3	640.5	486.4	294.1	234.8	346.4	28.35	16.93	69.44	41.46	9.277	13.97
		3	742	662.9	502.9	304.1	239	358.8	29.35	17.53	71.66	42.78	9.468	14.47
		5	770.1	693	525.3	317.6	244.8	375.3	30.59	18.26	74.9	44.72	9.670	14.98
		10	769.3	693.9	523.8	316.7	245.5	377.2	30.5	18.21	74.41	44.42	9.602	14.81
		20	771.2	697.2	524.8	317.3	246.4	379.9	30.57	18.25	74.76	44.64	9.703	15.06
	m=5	1	655.7	570.7	434.2	262.5	221.5	308.2	25.26	15.08	62.51	37.32	8.766	12.69
		2	696.5	613.8	466.6	282.1	229.7	331.7	27.15	16.21	66.92	39.95	9.083	13.48
		3	744.6	665.4	505.2	305.5	239.4	359.9	29.44	17.57	72.3	43.16	9.520	14.61
		5	729.6	649.2	493.1	298.2	236.4	351	28.71	17.14	70.64	42.18	9.379	14.24
		10	665.7	581.2	443.1	267.3	223.5	313.9	25.71	15.35	63.7	38.03	8.868	12.94
		20	668.3	584	444.3	268.6	224.1	315.4	25.84	15.42	64	38.2	8.887	12.99

Tab. 2 Results obtained by deterministic estimation, fuzzy clustering and approach based on fuzzy load flow

For total energy losses as well as losses of elements close to root node, approach based on the fuzzy load flow gives satisfactory results. The error of these results is within 5%. The error significantly increases with increasing of distance of element from the root node.

Accuracy of results obtained by fuzzy clustering depends on the number of clusters as well as on the value of coefficient *m*. The best results are obtained for *m* within interval 1.1-2. If coefficient *m* is larger then 2, accuracy decreases with increasing number of clusters. Results of fuzzy clustering for one cluster do not depend on the value of coefficient *m*, and results in these cases are lesser then real (correspond to calculation using mean values of loadings). Respecting the results shown in table 2, as well as the fact that number of clusters has minor influence on requested calculation time of fuzzy clustering, we can conclude that it is suitable to choose small value for *m* (e.g. 1.25) and 10-20 clusters.

5. CONCLUSION

Two approaches to calculating distribution energy losses are developed in this paper. First one is based on the fuzzy load flow, and second one on the fuzzy clustering technique. Methods respect real fact that loads (powers) for many load nodes are not exact known. Both approaches can calculate structure of losses (distribution of losses throughout the network elements).

Accuracy of the approach based on fuzzy load flow calculations depends on the chosen defuzzyfication method. For purposes of energy loss calculations the authors suggest bisector defuzzyfication method.

Accuracy of results obtained by fuzzy clustering depends on the number of clusters as well as on the value of coefficient *m*. The analyses made by authors are shown the best results are obtained for *m* within interval 1.1-2, and 10-20 clusters.

REFERENCES

[1] Арзамасцев Д. А., Липас А. В., Снижение технологического расхода энергии в электрических сетях, Высш. шк., Москва, 1989.

[2] Chang, R F, Leou, R C, Lu, C N: Distribution transformer load modeling using load research data, IEEE Trans. on Power Delivery, Vol. 17, No. 2, April 2002, pp. 655-661.

[3] Klir, G J, Yuan, B: Fuzzy sets and fuzzy logic: Theory and Application", 1995, Prentice Hall, New Jersey.

[4] Kuo, H C, Hsu, Y Y: Distribution system load estimation and service restoration using a fuzzy set approach, IEEE Trans. on Power Delivery, Vol 8, No. 4, October 1993.

[5] Rajaković, N, Stojanović, M, Tasić, D: An improved methods for the electric energy losses assessment in distribution networks, 3rd Mediterranean Conference Med Power 2002, Athens, November 4-6, 2002.

[6] Rajaković, N, Tasić, D, Stojanović, M: A clustering technique for distribution losses calculation in deregulated environment", proceeding of 2nd Balkan Power Conference, Beograd, Jun 19-21, 2002, pp. 31-34.

[7] Sarić, A, Ćirić, R: Integrated fuzzy state estimation and load flow analysis in distribution networks, IEEE Trans. on Power Delivery, Vol. 18, No. 2, April 2003.

[8] Shirmohammadi, D, Hong, H W, Semlyen, A, Luo, G X: A Compensation-based power method for weakly meshed distribution and transmission networks, IEEE Trans. On Power Systems, Vol. 3, No. 2, May, 1988, pp. 753-762.

[9] Stojanović, M, Tasić, D: A fuzzy method of distribution energy losses calculation, XXXVIII International Scientific Conference on Information and Energy Systems and Technologies, Sofia, October 2003, pp. 454-457.

[10] Воротницкий, В Э, Железко, Ю С, Казанцев, В.Н., Пекслис, В.Г., Файбисович, Д.Л.: Потери электроенергии в электрических сетях энергосистем, Энергоатомиздат, Москва, 1983.

[11] Железко Ю. С., Артемьев А. В., Севченко О. В., Расчет, анализ и нормирование потерь электроенергии в електрических сетях, НС. Энас, Москва, 2004.

Appendix 1: Fuzzy pseudopartition [3]

Let $X = \{x_1, x_2, ..., x_n\}$ be a set of given data. A fuzzy pseudopartition or fuzzy *c*-partition of *X* is a family of fuzzy subsets of *X*, denoted by $P = \{A_1, A_2, ..., A_c\}$ that satisfies:

$$\sum_{i=1}^{c} A_i(x_k) = 1, \qquad k \in N_n \ , \tag{A.1}$$

for $N_n = \{1,2,...,n\}$, and:

$$0 < \sum_{k=1}^{n} A_i(x_k) < n \qquad i \in N_c \ , \tag{A.2}$$

where *c* is positive integer and N_c set of integers $N_c = \{1,2,...,c\}$.

Appendix 2: Algorithm of fuzzy clustering

Algorithm of fuzzy clustering [3] is consisted of following steps:

Step 1.
Let $t = 0$. Select an initial fuzzy pseudopartition $P^{(0)}$.

Step 2.
Calculate the c cluster centers $v_1^{(t)}, ..., v_c^{(t)}$ by relation:

$$v_i = \frac{\sum_{k=1}^{n} [A_i(x_k)]^m x_k}{\sum_{k=1}^{n} [A_i(x_k)]^m} , \tag{A.3}$$

for $P^{(t)}$ and the chosen value of m.

Step 3.
Update $P^{(t+1)}$ by the following procedure: For each $x_k \in X$, if $\left\| x_k - v_i^{(t)} \right\|^2 > 0$ for all $i \in N_c$, then define:

$$A_i^{(t+1)}(x_k) = \left[\sum_{j=1}^{c} \left(\frac{\left\| x_k - v_i^{(t)} \right\|^2}{\left\| x_k - v_j^{(t)} \right\|^2} \right)^{\frac{1}{m-1}} \right]^{-1} , \tag{A.4}$$

if $\left\| x_k - v_i^{(t)} \right\|^2 = 0$ for some $i \in I \subseteq N_c$, then define $A_i^{(t+1)}(x_k)$ for $i \in I$ by any nonnegative real numbers satisfying:

$$\sum_{i \in I} A_i^{(t+1)}(x_k) = 1 , \tag{A.5}$$

and define $A_i^{(t+1)}(x_k) = 0$ for $i \in N_c - I$.

Step 4.
Compare $P^{(t)}$ and $P^{(t+1)}$. If $\left| P^{(t+1)} - P^{(t)} \right| \le \varepsilon$, then stop; otherwise, increase t by one and return to step 2.

In *Step 4*, $\left| P^{(t+1)} - P^{(t)} \right|$ denotes a distance between $P^{(t+1)}$ and $P^{(t)}$ in the space $R^{n \times c}$. An example of this distance is

$$\left| P^{(t+1)} - P^{(t)} \right| = \max_{i \in N_c,\, k \in N_n} \left| A_i^{(t+1)}(x_k) - A_i^{(t+1)}(x_k) \right| . \tag{A.6}$$

In the algorithm, the parameter is selected according to the problem under consideration. When $m \to 1$, the fuzzy c-means converges to a "generalized" classical c-means. When $m \to \infty$, all cluster centers tend towards the centroid of data set X. That is, the partition becomes fuzzier with increasing. Currently, there is no theoretical basis for an optimal choice for the value of m. However, it is established that the algorithm converges for any $m \in (1, \infty)$.

BIOGRAPHY

Dragan Tasić was born in Yugoslavia on 1961. He received the B.Sc. (1986) and Master of Science degrees (1991) from the University of Belgrade, and the Ph.D. degree from the University of Niš in 1997. He is now an Associate Professor in power systems at the University of Niš. His research interests include steady-state and dynamic analysis of power systems.

Miodrag Stojanović was born in Yugoslavia on 1972. He received his B.Sc. degree in electrical engineering, from the University of Belgrade, Yugoslavia in 1996 and M.Sc. degree (2003) from the University of Niš. His research interests include analysis of distribution systems and protective relaying.