SIMPLIFIED EQUIVALENT CIRCUIT OF ELECTRODYNAMIC FEEDER IN HEATING AND PUMPING REGIMES

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SUMMARY

Magnetohydrodynamic feeders of molten metals represent important devices for accurate dosing of conductive material. Design of these relatively expensive devices has to be based on reliable numerical models. Simulation of their behaviour in various operational regimes (heating, pumping) represents a complicated coupled task characterised by interaction of nonstationary electromagnetic, temperature and flow fields. Sometimes, however, their operation can be studied by means of the equivalent circuit representation. The paper deals with finding parameters of this circuit and their dependence on selected geometrical dimensions of the electrodynamic feeder for liquid zinc. The analysis is supplemented with an illustrative example and discussion of the results.

Keywords: *electrodynamic feeder, electromagnetic field, Joule losses, Lorentz forces*

1. INTRODUCTION

Optimised design of magnetohydrodynamic devices for transport, feeding or stirring [1] of liquid metals requires selection and realisation of numerical methods suitable for coupled analysis of electromagnetic, temperature and hydrodynamic fields. The task is complicated particularly in the case of magnetohydrodynamic feeders whose geometries are almost axi-symmetric, but all participating physical fields should be considered as 3D due to phenomena in the domain of the inlet and outlet of the device. In order to simplify the analysis, these phenomena are often neglected, so the task can be (with some error) considered as a 2D problem.

The feeder works in two regimes: heating and dosing. Because of the above reasons, their computer simulation is not an easy business [2], [3]. On the other hand, determination of its characteristic integral quantities (for instance total voltage of the supply resource or power factor) need not be based on this sophisticated field approach; methods using the equivalent circuit may in this case also provide sufficiently accurate results.

The paper deals with the electrodynamic feeder used for accurate dosing of refined liquid zinc. Analysis of 2D electromagnetic field in the channel provides distribution of the Lorentz forces for the regime of pumping, specific Joule losses for the regime of heating and also the Poynting vector [2], [3]. Knowledge of this vector in both regimes is then used for finding parameters of a simplified equivalent circuit of the device.

2. FORMULATION OF THE TECHNICAL PROBLEM

The basic disposition of the electrodynamic feeder for liquid zinc is in Fig. 1. The feeder consists of three parts: a ring-shaped container with liquid metal, inlet and outlet, three field coils carrying harmonic currents and magnetic core. The magnetic core consists of the bottom part and limb.

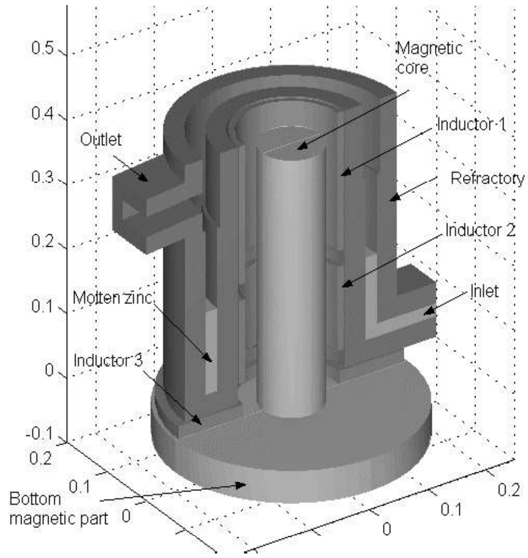

Fig. 1 Basic disposition of the electrodynamic feeder

The feeder works by turns in two regimes: heating and dosing. The purpose of the heating regime is to keep metal in liquid state between two dosing periods. This is realised by inductors 1 and 2. The heating power due to corresponding Joule losses in metal should cover losses caused by heat conduction and convection (influence of radiation does not play any decisive role due to low temperature conductivity of the refractory). During the period of heating no significant vertical electromagnetic forces are exerted in the molten zinc and its level remains at the bottom position deep below the outlet. The dosing regime, on the other hand, is characterised by co-operation of inductors 2 and 3. Eddy currents induced in molten zinc together with the magnetic flux density produce electromagnetic forces with a significant vertical component. As long as these forces are sufficiently high, they make liquid material rise above the level of the outlet and the metal is pumped through it out of the container. During this regime zinc is continually fed into the container through the inlet from a near basin.

The task is to investigate the situation in the channel in both described regimes and find parameters of the corresponding equivalent circuit of the device.

3. MATHEMATICAL MODEL OF THE PROBLEM

In order to simplify the analysis, phenomena in the domain of the feeder inlet and outlet are neglected and the task is handled as 2D axi-symmetric. The simplified axial cut through the feeder is shown in Fig. 2.

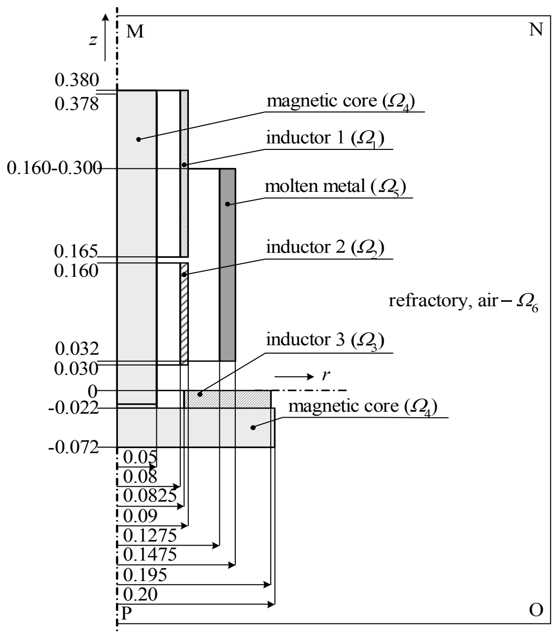

Fig. 2 Simplified axial cut through the feeder

The definition area Ω of the problem consists of six subdomains Ω_1–Ω_6 with different physical properties. All three inductors Ω_1–Ω_3 are supposed to carry harmonic currents of given amplitudes I_1, I_2, I_3 and industrial frequency $f = 50$ Hz (such a low frequency allows neglecting the skin effect within them). Subdomain Ω_5 representing the molten metal of permeability μ_0 and electrical conductivity γ_m changes with its variable level. Subdomain Ω_6 involves the ceramic parts (refractory) and air ($\mu_r = 1$, $\gamma = 0$). Relative permeability of the magnetic core Ω_4 is considered constant, so that distribution of the magnetic field in Ω can be described by the Helmholtz equation for the phasor of magnetic vector potential $\boldsymbol{A}$ in the form [4]

$$\mathrm{rot\,rot}\,\underline{\boldsymbol{A}} + \mathrm{j}\cdot\omega\mu\gamma\underline{\boldsymbol{A}} = \mu\underline{\boldsymbol{J}}_{\mathrm{ext}} \tag{1}$$

where μ is the magnetic permeability, γ the electrical conductivity, $\underline{\boldsymbol{J}}_{\mathrm{ext}}$ the actual homogeneous density of the external currents in the inductors and $\omega = 2\pi f$. The boundary condition along an artificial boundary MNOPM placed at a sufficient distance from the feeder reads

$$\underline{\boldsymbol{A}} = \boldsymbol{0}\,. \tag{2}$$

The harmonic magnetic field produces in molten metal eddy currents of density $\boldsymbol{J}$ whose phasor is given as

$$\underline{\boldsymbol{J}} = \mathrm{j}\cdot\omega\gamma\underline{\boldsymbol{A}}. \tag{3}$$

The specific Joule losses Q and Lorentz forces $\boldsymbol{f}$ due to these currents are

$$Q = \gamma\omega^2\left|\underline{\boldsymbol{A}}\right|^2, \tag{4}$$

$$\underline{\boldsymbol{f}} = \underline{\boldsymbol{J}} \times \underline{\boldsymbol{B}} = \underline{\boldsymbol{J}} \times \mathrm{rot}\,\underline{\boldsymbol{A}} \tag{5}$$

provided that $\boldsymbol{J}$ and $\boldsymbol{A}$ denote the effective values of both quantities. The molten metal is levitated and pumped out only when the pressure caused by the axial component of the Lorentz force is at any instant higher than the corresponding pressure due to the gravitational force. The total Joule losses and all components of electromagnetic force are obtained by integrating the above quantities over the metal area.

In fact, the field current density $\boldsymbol{J}_{\mathrm{ext}}$, vector potential $\boldsymbol{A}$ and eddy current density $\boldsymbol{J}$ have only one nonzero component ($J_{\mathrm{ext},\alpha}$, A_α, J_α) in the tangential direction, which leads to substantial simplification of (1), (3), (4) and (5).

The simplified equivalent circuit of the feeder is depicted in Fig. 3. It can be described by the phasor equation

$$\underline{E} = R_{\mathrm{S}}\underline{I} + \mathrm{j}\cdot\omega L_{\mathrm{S}}\underline{I} + \underline{Z}\,\underline{I} \tag{6}$$

where E is the electromotive force, R_{S} and L_{S} are the resistance and inductance of the source and feeding circuit and $\underline{Z}$ is the total impedance of the feeder. This impedance can be computed from the known distribution of electromagnetic field by integrating the normal component of the Poynting vector over the boundaries Γ_{c} of all inductors and dividing the result by the squared current.

$$\underline{Z} = \frac{\oint_{\Gamma_{\mathrm{c}}}\left(\underline{\boldsymbol{E}} \times \underline{\boldsymbol{H}}^{*}\right)\cdot \mathrm{d}\boldsymbol{n}}{\underline{I}^{2}} \tag{7}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are vectors of electric and magnetic field strength, respectively, while $\boldsymbol{n}$ is the vector of the outward normal.

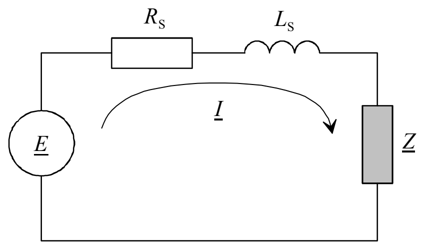

Fig. 3 Simplified equivalent circuit

Because of linearity of the model the integral in the above formula is exactly proportional to the square of current so that the impedance of the feeder is independent of it. Therefore, the impedance can be computed using any feeding current while the real current follows directly from (6).

4. ILLUSTRATIVE EXAMPLE

Both regimes were modelled on a feeder of molten zinc designed and manufactured in the Department of Electrotechnology of the Silesian University of Technology, Katowice, Poland [5]. Its principal dimensions in the simplified arrangement are given in Fig. 2 (width w of the container may increase with its growing external radius). Other important parameters follow:

Inductor 1, 2 and 3: hollow copper conductor of internal radius 3 mm and external radius 4 mm. Number of turns of the first two inductors N_1 = 20, N_2 = 12, inductor 3 has 2 layers by N_3 = 13 turns in each layer.

The magnetic core is laminated (thickness of magnetic sheets is 0.35 mm), with relative permeability μ_r = 1000.

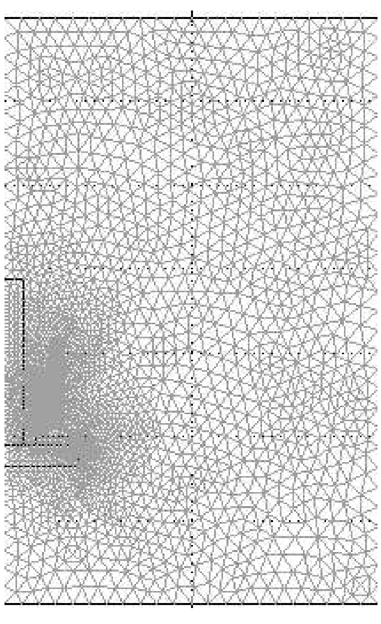

Fig. 4 The first rougher mesh

Liquid zinc: density ρ = 6690 kg/m^3, electrical conductivity γ_m = 3.4 MS/m, thermal conductivity λ_m = 116 W/m K, specific heat c = 0.394 J/kg K, kinematic viscosity $\eta_k = 421\cdot10^{-7}$ m^2/s.

Calculations were carried out by Femlab 2.3 and Matlab in combination with procedures developed and written by the author. Geometrical convergence of the results was tested on two triangular meshes with different roughness. Compared were the total Joule losses Q in zinc and also the total axial force F_z acting there. Computations were performed for the pumping regime.

The first mesh is depicted in Fig. 4. The second mesh was generated from the first mesh by uniform refining in all subdomains. The results are summarised in Tab. 1.

Table 1: Geometrical convergence of results

number of nodes	4083	16203
number of elements	8038	32152
number of degrees of freedom	16203	64557
maximum area of the element (cm^2)	0.00125481	0.000313701
total losses Q (W)	463.92	463.98
total axial force F_z (N)	21.181	21.184

It was proved that for meshes exceeding about 30000 elements the field quantities at selected points changed in maximum by tenths of percent. An analogous verification was carried out for the location of the artificial boundary MNOPM.

Computational time of one task (distribution of the field and forces) took about 1 min.

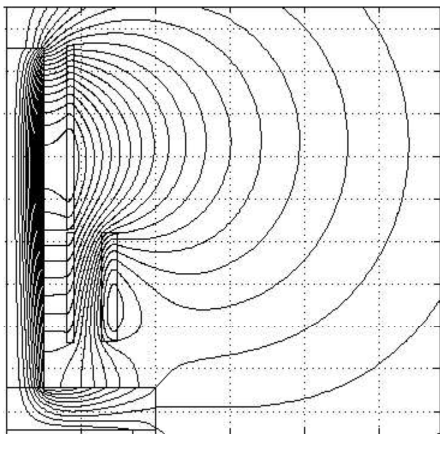

Fig. 5 Distribution of the magnetic field in the regime of heating (low level of zinc: h = 0.16 m)

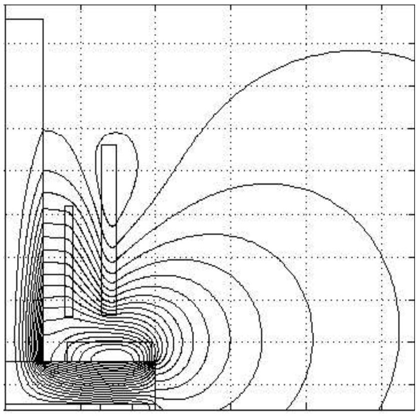

Fig. 6 Distribution of the magnetic field in the regime of pumping (average level of zinc: $h = 0.23$ m)

Solution of the task provided a lot of results. Some of them are shown and discussed in the next text. Fig. 5 shows the distribution of magnetic field in the arrangement in the regime of heating (inductors 1 and 2 are on, $I_1 = I_2 = 100$ A). On the other hand, Fig. 6 shows an analogous distribution in the regime of pumping (coils 2 and 3 on, $I_2 = I_3 = 100$ A), when the level of zinc is at its average value.

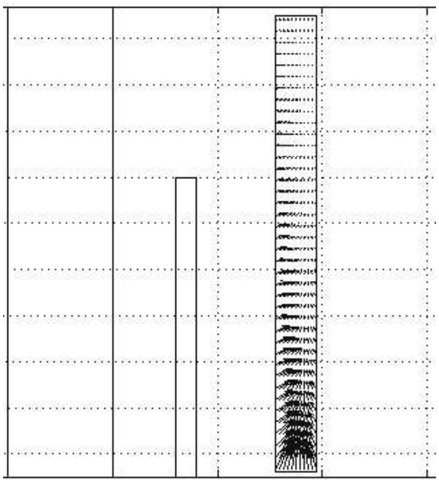

Fig. 7 Distribution of the local forces in the channel with zinc ($h = 0.23$ m)

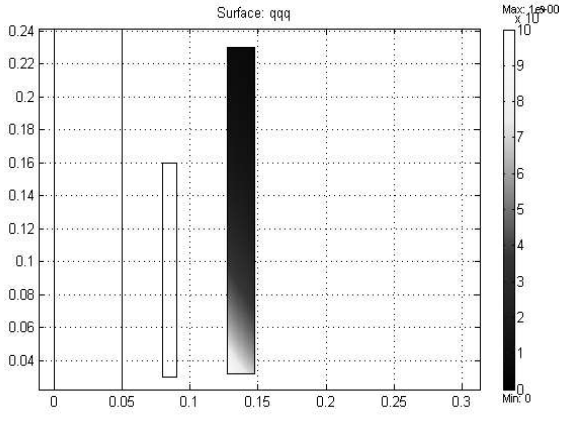

Fig. 8 Distribution of the specific Joule losses in zinc

For this regime (height of the zinc level h being 0.23 m) of pumping Fig. 7 shows distribution of the local forces in zinc and Fig. 8 distribution of the specific Joule losses due to eddy currents. It can be seen that the values of both quantities reach their maxima in the bottom part of melt, which is caused by strong magnetic field generated by field coil 3.

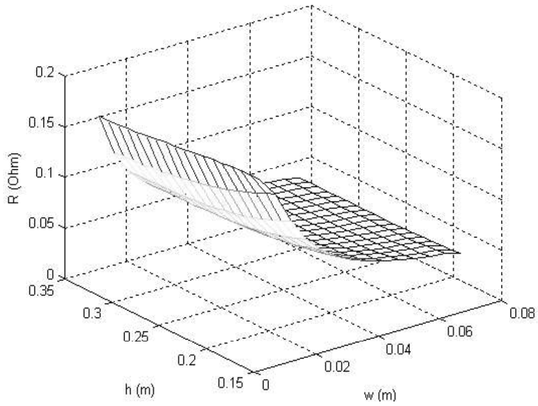

Fig. 9 Dependence of the equivalent resistance of the feeder on width w and height h of the channel

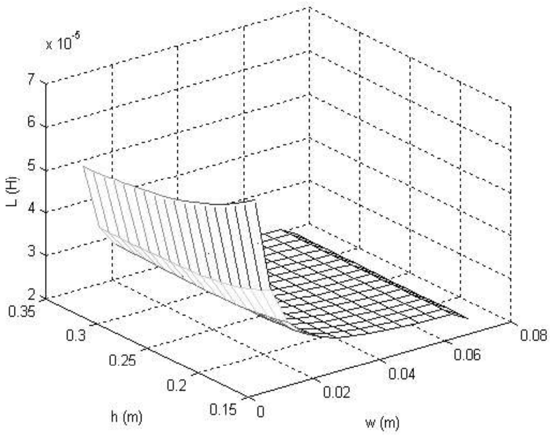

Fig. 10 Dependence of the equivalent inductance of the feeder on width w and height h of the channel

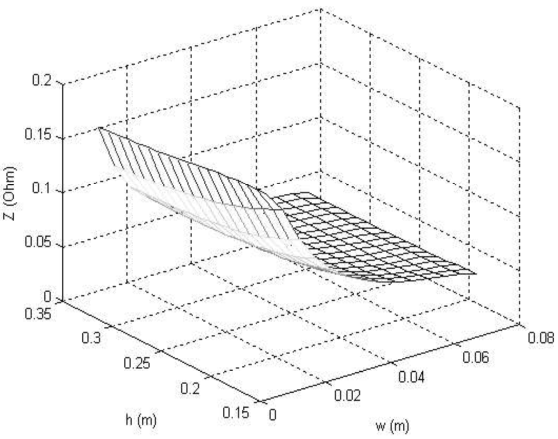

Fig. 11 Dependence of the equivalent impedance of the feeder on width w and height h of the channel

Distribution of the magnetic field as well as the forces and losses in the channel substantially depends on its geometry. Its width w can grow in the direction of its external radius (such modification of the channel does not represent any technological problem) while its height h is given by the instantaneous position of the zinc level. A series of further computations based on relation (7) provided dependencies of the equivalent impedance of the feeder on the above dimensions. Of course, in the regime of pumping the method gives only approximate results because it does not respect loss of energy due to motion of the zinc column. More accurate results we obtain for the regime of heating. Figs. 9, 10 and 11 show the equivalent resistance, inductance and impedance of the device as functions of geometrical parameters w and h. The narrower is the channel, the higher are all these quantities.

5. CONCLUSION

The electromagnetic field in a zinc feeder both in the regime of dosing and heating has been modelled. The Joule losses and electromagnetic forces have been computed. All these computations have been carried out for various geometrical parameters of the channel with zinc so that dependencies on these parameters could be obtained and depicted. In addition, a model of a simple linear feeding circuit has been suggested.

The next work should take into consideration using of more advanced calculation method making possible to receive all parameters of the equivalent circuit. It should be also supplemented by procedures of solving flow field, which would allow establishing distribution of velocity in the liquid metal and then a resulting flow-rate.

ACKNOWLEDGEMENT

Financial support of the Grant Agency of the Czech Republic (Project No. 102/03/0047) is gratefully acknowledged.

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BIOGRAPHY

Ladislav Musil obtained the degree Ing. from the Faculty of Electrical Engineering in 2002 and just after started his doctoral studies in the Department of Electrical Power Engineering. He deals with the mathematical and computer modelling of coupled problems, particularly in heavy current applications. He is an author or co-author of about 15 papers in the area.