MECHANICAL CALCULATION OF OVERHEAD POWER LINE CONDUCTOR UNDER COMBINED LOAD

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SUMMARY

This paper deals with mechanical solution of overhead transmission lines under combined (ice and concentrated mechanical load), functioning in vertical direction. Line conductor under non-equally distributed loads along the span is analysed as combined, i.e. inhomogeneous elastic thread, that is not continuous parabola along whole span. For the sake of the use of conductor state equation, inhomogeneous mechanical load must be replaced by equivalent homogenous load by means of the load factor. To solve of these problems allows so-called " method of equivalent support". The basic principle of this method is substitution of conductor under optional loading by equivalent elementary one-distance hinge jointed support loaded as conductor. By this method (by destination of support diffracted moments from equations of equilibrium) is possible common determination of expressions for the calculation of mechanical parameters of conductor under combined loading. Theoretical analysis, i.e. sequence of venting relations for determination of shape of suspended conductor, mechanical stress, loading factor, equivalent mechanical load, length of conductor and its sag in arbitrary point on level span for common combination loading is complemented by example for concrete, us defined combined loads. Concrete, ice coating-one concentrated load and ice coating-three concentrated load. The example of concentrated load are interphase spacers used for increase of safety of overhead transmission lines by elimination of danger touching of conductors or by elimination of undesirable consequences of dynamic effects at the line. Given method can be used for designing and installation of this spacers, as well as to determination of result static state and the result changes of mechanical parameters of line after their installing.

Keywords: *parabola, concentrated load, sag, tension of conductor, support.*

1. INDRODUCTION

Mechanical parameters of overhead transmission lines with spans less than 400m is possible to solve by the approximate methods. Approximate methods suppose suspended conductor as elastic homogenous thread with the parabolic shape.

Conductor under non-uniform distributed loads along the span must by analysed as combined, i.e. inhomogeneous elastic thread, that is not continuous parabola along whole span. Discontinuity of curve is on the meeting places with different size of load and in concentrate loads locations, too.

The form of state equation for inhomogeneous load, basically is the same as for homogenous load, but inhomogeneous load must be replaced by equivalent homogenous load

$$q_{\text{ekv}} = \frac{q \cdot K}{\cos\delta} \tag{1}$$

where q_{ekv} - equivalent vertical load per unit conductor length [N/m], q - vertical load per unit homogenous conductor length [N/m], K – factor of loading determined from equality of homogenous and inhomogeneous wire length (see chapter 2.2), δ [°]- elevation angle of the span.

The length of inhomogeneous conductor must be determined by integrating per parts with continuous shape of parabola curve.

2. METHOD OF EQUIVALENT SUPPORT

To develop a solution are made following assumptions:

- absolute zero point is in point A,
- x-axis is positive directed on the right,
- y-axis is positive down directed,
- $q(x)$ is common non-uniform load distributed continually along the span A-B,
- $P_{1\dots}$ P_n are concentrated mechanical loads distributed along the span A-B,
- loads $q(x)$ and $P_{1\dots}$ P_n affect in vertical direction, it comes to this, this analyse is solved in „*xy*" plane.

2.1 Span of conductor under non-uniform load

Figure 1a. shows conductor loaded by non-uniform $q(x)$ and concentrated $P_{1\dots}$ P_n loads, below (Figure 1b.) is support loaded as conductor, where:
a - span of suspended conductor [m],
f(x) - sag of conductor in space x [m],
F_A , F_B - resultant forces in points A, B [N],
F_{HA} , F_{HB} - horizontal parts of forces F_A , F_B [N],
F_{VA} ,F_{VB} -vertical parts of forces F_A , F_B [N].

From equilibrium equations (2), (3), (4)

$$\sum F_X = -F_{HA} + F_{HB} = 0 \tag{2}$$

$$\sum M_{yA} = -F_{VB} \cdot a + \sum M_{yqA} + \sum M_{yPA} = 0 \tag{3}$$

$$\sum M_{\mathrm{yB}} = -F_{\mathrm{VA}} \cdot a + \sum M_{\mathrm{yqB}} + \sum M_{\mathrm{yPB}} = 0 \tag{4}$$

are results given by expressions (5), (6), (7):

$$F_{\mathrm{HA}} = F_{\mathrm{HB}} = F_{\mathrm{H}} \tag{5}$$

$$F_{\mathrm{VA}} = \frac{\sum M_{\mathrm{yqB}} + \sum M_{\mathrm{yPB}}}{a} \tag{6}$$

$$F_{\mathrm{VB}} = \frac{\sum M_{\mathrm{yqA}} + \sum M_{\mathrm{yPA}}}{a} \tag{7}$$

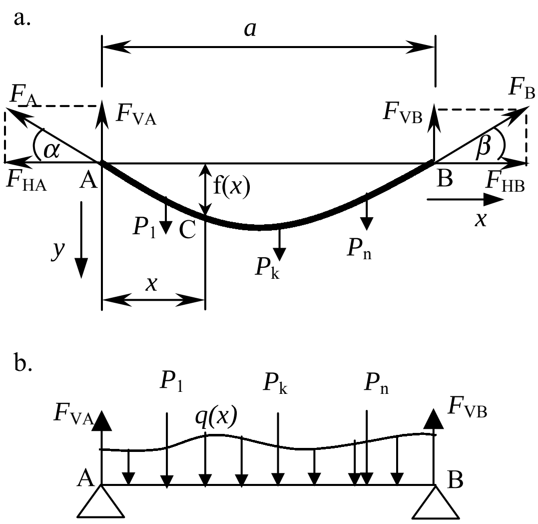

Fig. 1 a. Conductor loaded by common non-uniform *q(x)* and concentrated P_{i} loads
b. Equivalent support loaded as conductor

where:
F_{H} - horizontal force in lowest point of conductor,
M_{yqA}, M_{yPA} (M_{qB}, M_{PB}) - moments of all loads action related to the point A (B).

Equilibrium equation at any point of conductor (point C - Figure1), for all – from left - applied moment action, is

$$\sum M_{\mathrm{C}} = \underbrace{F_{\mathrm{VA}} \cdot x - \sum M_{\mathrm{qC}} - \sum M_{\mathrm{PC}}}_{M(x)} - F_{\mathrm{H}} \cdot y = 0 \tag{8}$$

Algebraic sum of moments of vertical forces in this equation equals diffracted moment, that is created on the elementary one-distance hinge jointed support, loaded as a conductor. If this support moment is specified by $M(x)$, then

$$\mathrm{f(x)} = \frac{M(x)}{F_{\mathrm{H}}} \tag{9}$$

Expression (9) can be used to determine of the sag of conductor in any position of the span.

2.2 Length of conductor under non-uniform load

Length of non-uniform loaded conductor must be determined as summation of the part lengths with continuous behaviour of the parabolic curve.
Differentiation of equation (9) is:

$$\frac{dy}{dx} = \frac{1}{F_{\mathrm{H}}} \cdot \frac{dM(x)}{dx} = \frac{Q(x)}{F_{\mathrm{H}}} \tag{10}$$

and element of conductor length is:

$$dL = dx \cdot \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \tag{11}$$

By binomial decomposition and by integration of (11) is given expression for calculation of conductor length, i.e.

$$L = \int_0^a dx + \frac{1}{2F_{\mathrm{H}}^2} \int_0^a Q^2(x)dx - \frac{1}{8F_{\mathrm{H}}^4} \int_0^a Q^4(x)dx + \\ + \frac{1}{16F_{\mathrm{H}}^6} \int_0^a Q^6(x)dx - \ldots \tag{12}$$

With sufficient precision of calculation we can applied only first and second part of the past equation [2], i.e.

$$L = \int_0^a dx + \frac{1}{2F_{\mathrm{H}}^2} \cdot D \tag{13}$$

where

$$D = \int_0^a Q^2(x)dx \tag{14}$$

Equation (14) can be used for determination of load factor *K*. With assumption of equality of uniform and non-uniform loaded conductor length, i.e. $L_{\text{non-unif}} = L_{\text{unif}}$, expression (14) must be modified to the form $D_{\text{non-unif}} = D_{\text{unif}} \cdot K$.

3. APPLICATION OF ANALYSIS

Figure 2 shows conductor loaded by one concentrated force at the span A-B. By the use of previous method of equivalent support, is possible to determine concrete equations for calculate of conductor sag in any position at the span and length of conductor under concentrated load.

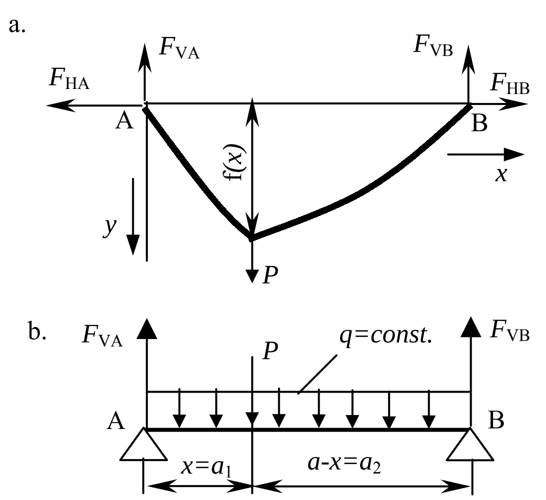

Fig. 2 a. Conductor loaded by common non-uniform $q(x)$ (in this case q=const.) and one concentrated load P
b. Equivalent support loaded as conductor

Vertical forces in handing points or bearing forces of equivalent support F_{VA}, F_{VB} are given by equations:

$$F_{VA} = \frac{q.a}{2} + \frac{P.a_2}{a} \tag{15}$$

$$F_{VB} = \frac{q.a}{2} + \frac{P.a_1}{a} \tag{16}$$

If we can to determine sag of conductor loaded by one concentrated force in any position at the span, we must to separate the span a in two intervals $(0;\ a_1)$ and $(a_1;\ a)$. In these intervals is kept continuous parabolic shape of conductor. In interval $(0;\ a_1)$ is support deflection moment related to point $x \in (0;\ a_1)$, caused by all left forces, determine by expression:

$$M(x) = F_{VA}.x - q.\frac{x^2}{2} \tag{17}$$

It comes to it, after substitute (17) to (9) is possible obtain equation of shape of conductor loaded by one concentrated force P for interval $(0;\ a_1)$. In like manner, for interval $(a_1;\ a)$:

$$M(x) = F_{VA}.x - q.\frac{x^2}{2} - P.(x - a_1) \tag{18}$$

Shape of suspended conductor in interval $(a_1;\ a)$ we can get by substitute (18) to (9).

To obtain length of conductor, is necessary determine value of integral D as summation particular integrals in intervals $(0;\ a_1)$ and $(a_1;\ a)$, i. e.:

$$D = D_I + D_{II} = \int_0^{a_1} Q_I^2(x)dx + \int_{a_1}^{a} Q_{II}^2(x)dx = \int_0^{a_1}(F_{VA} - q.x)^2 dx + \int_{a_1}^{a}(F_{VA} - P - q.x)^2 dx \tag{19}$$

After substitute (14) instead F_{VA} and by the reform of this expression we obtain:

$$D = \frac{q^2.a^3}{12}.\left(1 + 12.\frac{P.a_1.a_2}{q.a^3} + 12.\left(\frac{P}{q.a}\right)^2.\frac{a_1.a_2}{a^2}\right) = \frac{q^2.a^3}{12}.K^2 = \frac{q_{ekv}^2.a^3}{12} \tag{20}$$

Then length of conductor is:

$$L = a + \frac{1}{2F_H^2}\cdot\frac{q^2.a^3}{12}\cdot 1 + \left(12.\frac{P.a_1.a_2}{q.a^3} + 12\left(\frac{P}{q.a}\right)^2.\frac{a_1.a_2}{a^2}\right) \tag{21}$$

The value of horizontal force F_H we can obtain from equation of state [1], where is necessary to consider equivalent uniform loading q_{ekv} given by equation:

$$q_{ekv} = \sqrt{\left(1 + 12.\frac{P.a_1.a_2}{q.a^3} + 12\left(\frac{P}{q.a}\right)^2.\frac{a_1.a_2}{a^2}\right)}\cdot q \tag{22}$$

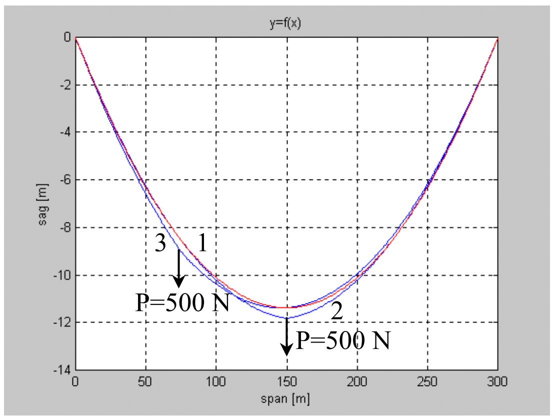

Fig. 3 Shapes of curves of conductor AlFe 240/39 in the level span under ice load (16,80 $N.m^{-1}$):
1. without concentrated load
2. with concentrated load 500 N located in the middle of span
3. with one concentrated load located in space x=75 m

For the sake of vision about changes of parameters of conductor loaded by concentrate load, was calculated:

- mechanical tension,
- equivalent uniform load,
- static profile,
- length

of conductor AlFe 240/39 (q_1=9,656 N.m^{-1}), suspended on the level span a =300 m for variable location of concentrated load 500 N (see Tab.1, Tab.2, Fig.3, Fig.4), ambient temperature is -5°C.

	without concentra - ted load	with concentrated load	
		a_1=75m	a_2=150m
K [-]	1	1,2023	1,2625
q_{ekv} [N.m^{-1}]	9,656	11,610	12,192
σ_H [MPa]	92,886	98,728	100,450
l [m]	301,15	301,17	301,18

Tab.1 Mechanical parameters of conductor under combined load (see Fig.3)

Figure 4 shows the same conductor loaded by ice coating (q_2=16,80 N.m^{-1}) and three concentrated load concurrently located along the level span.

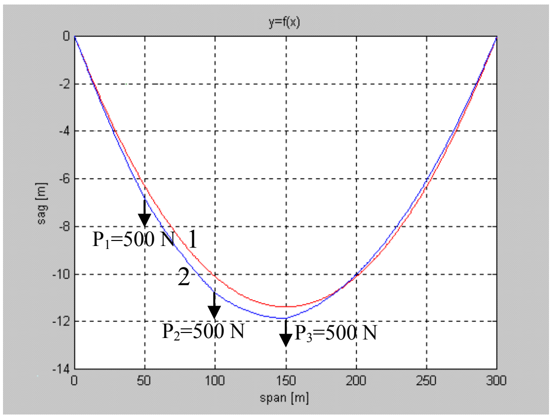

Fig. 4 Shapes of curves of suspend conductor AlFe 240/39 in level span 300m with ice load:
1. without concentrated load
2. with three concentrated load

	without concentrated load	With three concentrated load
K [-]	1	1,6433
q_{ekv} [N.m^{-1}]	9,656	15,869
σ_H [MPa]	92,886	111,160
l [m]	301,15	301,23

Tab.2 Mechanical parameters of conductor under combined load (see Fig.4)

4. CONCLUSIONS

This paper deals with effect of combined (concentrated and ice loading) of overhead transmission lines. One of example of concentrated load are interphase spacers used for increase of safety of overhead transmission lines by elimination of danger touching of conductors or by elimination of undesirable consequences of dynamics effect at the line. Given method can be used for designing and installation of this spacers, as well as to determination of result static state and the result changes of mechanical parameters of line conductor after their installing.

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BIOGRAPHY

Ladislav Varga was born in 1946 in Košice. He received his Ing degrees in the Department of Electro-Power Engineering of the Technical University in Košice (1967), and CSc (PhD) degrees in the Department of Electro-Power Engineering of the Technical University in Bratislava (1977). He is Associate Professor at the Department of Electro-Power Engineering of the Technical University of Košice. His research interests include mechanical solution of overhead power lines and design of large grounding systems.

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